The Effect of Macroeconomic Volatility on Financial Deepening: A Missing Link?

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Abstract

As is well-documented, most emerging countries exhibit higher and more persistent output volatility than developed countries. Many empirical and theoretical works identify the shallow financial system as the source of output volatility, resulting in inefficient risk-sharing and credit misallocation. Other studies, however, emphasize that patterns of production and the lack of technological diversification are responsible for the high and persistent volatility, but without considering that this volatility may hinder the development of financial system. To explore the latter interaction, I develop a general equilibrium framework to explain the channels through which high and persistent macroeconomic volatility can affect financial deepening, by focusing on the exogenous shocks originating in the real sector. I find that the volatility of total factor productivity (TFP) shocks does not translate one-to-one to a change in output volatility, because of adjustments to investment in capital accumulation induced by changes in the risk premium. This response of the risk premium to TFP shocks is the missing link explaining why real volatility can hinder financial deepening. A shallow financial system can thus be a by-product of the high macroeconomic volatility. Since implementing policies to create a more resilient output sector requires time and resources, in the meantime, policy makers should focus on reducing macroeconomic fluctuations to trigger its beneficial effects on financial development.

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1 Introduction

Most emerging economies have exhibited higher and more persistent macroeconomic volatility than developed countries, a stylized fact that has been well documented by the literature.[1] The debate has concentrated on the sources of such volatility. An important strand of the theoretical and empirical literature argues that the shallow financial system in less developed countries is one of the main factors explaining their output volatility.[2] The theoretical arguments rely on two transmission mechanisms. First, instead of absorbing the shocks, credit market imperfections can amplify a small exogenous shock.[3] In addition, a lack of opportunities for financial diversification means risk-averse agents avoid investing in riskier projects with higher productivity (returns), and instead invest in projects with low risk.[4] An alternative view is that the macroeconomic volatility observed in many emerging countries originates in their production patterns, which are highly concentrated in more volatile industries with non-flexible and less diversified technologies.[5] The question arises whether these production patterns and the related volatility hinder the development of the financial sector, possibly explaining the shallow financial system.

In this chapter, I follow the second strand of literature where the macroeconomic volatility is an *intrinsic phenomenon* originating in the real sector with independence of the initial level of financial development (deepening).[6] Therefore, if the financial deepening is not the primary source of excess volatility, the question remains of how the high and persistent macroeconomic fluctuations might affect the development of a financial system. Although some intuitive answers could be given with a partial equilibrium framework, the question requires a general equilibrium approach to explain the causal relationship between the macroeconomic volatility and the financial system. To this aim, I propose a general equilibrium framework to study the channels through which the persistent aggregate volatility can affect the financial deepening, proxied by the private credit to GDP ratio.[7]

[1] Among other results, Aguiar and Gopinath (2007) find the income level and growth in emerging markets are twice as volatile as those in developed economies for the period 1980-2003.

[2] The empirical studies have found mixed evidence. Among others, Braun and Larrain (2005) and Raddatz (2006), show that the financial development can reduce output volatility, while Wang et al. (2018) find that its effectiveness is decreasing. However, Acemoglu et al. (2003) argue that institutions (instead of financial development) can explain not only volatility but also growth and crisis. Beck et al. (2006) do not find a robust relationship between financial intermediary development and growth volatility, when they control with terms of trade shocks and inflation.

[3] See Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Aghion et al. (2010).

[4] See Greenwood and Jovanovic (1990), Acemoglu and Zilibotti (1997), and Aghion et al. (1999)

[5] See Koren and Tenreyro (2007),Kraay and Ventura (2007), and Koren and Tenreyro (2013).

[6] Koren and Tenreyro (2013) show that the decline in aggregate volatility is independent of the country's financial development.

[7] The level of financial development (or deepening) is generally proxied by the ratio of private credit

The model is an extension of Labadie (1998) with an optimal contract scheme like the one in Bernanke, Gertler, and Gilchrist (1999). There are two types of firms, the producers of consumption goods and the producers of capital goods (investors). Consumption producers are directly exposed to total factor productivity (TFP) shocks, while capital producers are subject to an uninsurable idiosyncratic risk whose distribution is affected by the aggregate volatility. Throughout this chapter, the standard deviation of TFP shocks is referred as *aggregate risk*, since it cannot be diversified away. In addition, the financial intermediary diversifies its portfolio with two possible investment options: lending to the capital producer or buying a risk-free government bond. The realization of the idiosyncratic shock is private information known only by capital producers, but observable to the financial intermediary if he pays a monitoring cost, i.e. a costly state verification (CSV) approach.

Although the interdependence between macroeconomic fluctuations and producer-specific shocks has been studied to explain investment decisions and leverage, the novelty of my approach is to study how this interdependence affects not only the investment decisions, but also the output volatility and financial deepening. Another important feature of the model is that the aggregate risk (TFP volatility) does not have a one-to-one effect on the output volatility (variance of the aggregate output), since it is partially offset by the response of investment to changes in the aggregate uncertainty. This finding, however, depends on the elasticity of the risk premium with respect to the aggregate risk.

The main results of the chapter can be summarized as follows: (i) A reduction (increase) in the aggregate volatility increases (reduces) the optimal level of private loans and investment by lowering (increasing) the risk premium of lending to the capital producers; (ii) A corollary is that the amount of domestic loanable funds available to the government is positively correlated with the aggregate volatility, that is, the government can sell more bonds to the domestic financial system as real volatility increases; (iii) The reduction in the output volatility is lower than the decrease in aggregate risk, given the impact of the latter on investment, depending on the risk-premium elasticity with respect to aggregate uncertainty; (iv) Finally, as the aggregate volatility decreases the financial depth indicator (FD) may increase. While most empirical and theoretical works discuss the causal effects of FD on the output volatility, this is one of the first studies that addresses the channels to uncover the reverse causality through a simplified general equilibrium framework. In sum, if a country faces intrinsic volatility from the real sector, then the underdeveloped financial system can be a by-product of that volatility.

In countries with this type of volatility, the challenges from the policy makers' perspec-

to GDP, while most of the theoretical models narrow even further the concept, focusing on investment in fixed capital as the main purpose of borrowing. Having a single indicator to represent the complexity of the financial development can generate misleading measures of the actual financial depth and access (Čihák et al., 2012; Sahay et al., 2015).

tive would be how to reduce the aggregate fluctuations. To attack the source of this problem, moving to technologies with greater resilience to shocks and diversifying production would be a reasonable solution in the long-run. However, a change in the production patterns with technological diversification requires time and resources, particularly in countries where the financial system is endogenously shallow. Additionally, economic agents must deal with the intrinsic volatility in the short and medium run. Therefore, in the meantime, policy makers should focus in reducing the aggregate volatility, for example by applying more counter-cyclical macroeconomic policies, to lower the risk of investment projects and increase the amount of credits to the private sector.[8]

The chapter is organized as follows. Section 2 provides the main features of the benchmark economy without aggregate volatility, but with idiosyncratic risk. In the following sections, the aggregate uncertainty is introduced in the form of TFP shocks to study how different levels of aggregate risk can affect the macroeconomic fluctuations and the financial depth indicator. Finally, some conclusions and possible extensions are summarized.

2 The Benchmark Model: Deterministic Case

The economy has two goods, a multi-purpose consumption good and a capital good. There are five types of agents: two-period lived households, producers of consumption goods, two-period lived producers of capital goods, and infinitely lived financial intermediaries and government.

In each period, there are N_t young households and N_{t-1} old households. For simplicity, I assume no population growth,i.e. $N_{t-1} = N_t = N$. Young households are endowed with L units of labor, *normalized to unity*, that is inelastically supply to the consumption-good producers at a wage rate. With the labor income, young agents consume, pay a lump-sum tax, and save in the form of deposits with the financial intermediary. Old agents only consume by using their savings when young. Producers of consumption goods use both capital goods and labor, renting capital from the capital-producers, who have access to the investment technology. In order to invest, they borrow from the financial intermediary, who decides the composition of its portfolio between lending to the capital producer and buying a risk-free government bond. In this section, there is only an idiosyncratic shock that affects the return of capital-producers investment. The realization of this specific shock is only known by each producer in the second period of its life. However, this information is observable to the financial intermediaries at a monitoring cost, in other words, a costly state verification (CSV) assumption.[9] The government simply spends all the tax revenue and the

[8]See Calderón, Chuhan-Pole, and López-Monti (2017) for an assessment of the cyclical properties of fiscal policies from the expenditure side in Sub-Saharan Africa and other developing regions

[9]Having two production sectors is not only a strategy to separate the investment decision, but also to

funds offered by the financial intermediaries net of debt repayment. Next, the benchmark model is explained in detail.

2.1 Consumption-Good Producers

There is a unit mass of producers of consumption goods, each of whom rents capital (K_t) from the capital producers, which completely depreciates in one period, and hires the young agents (N) to produce Y_t units each period. The Cobb-Douglas function is given by $Y(K_t, N) = AK_t^{\delta} N^{(1-\delta)}$. There are no aggregate shocks in this benchmark case, thus, without a loss of generality, total factor productivity (TFP) is assumed to be constant over time (A).

Given competitive markets, each factor is paid its marginal product, so the per-worker returns $w(k_t)$ and $\rho(k_t)$ are the wage rate and the return of capital respectively:

$$w(k_t) = (1-\delta) A k_t^{\delta}, \tag{1}$$

$$\rho(k_t) = \delta A k_t^{(\delta-1)}, \tag{2}$$

where $k_t = K_t/N_t$. In the first period ($t = 1$), each producer has an initial stock of capital k_0.

2.2 Households

In the first period, new-born agents (*subscript 1*) inelastically supply labor at wage w_t. They decide how much to consume $c_{1,t}$ and save in the form of deposits (d_t^s) after paying a lump-sum tax to the government (τ_t). Individual deposits can be thought as equity shares and the return as dividends. When agents are old (*subscript 2*), they consume their deposits including a gross deposit rate $r^d(k_{t+1})$. Since the young generations are the ultimate owners of all financial institutions, the return on deposits can be thought as dividends paid on equity shares d_t. Taking as given prices $(w(k_t), r^d(k_{t+1}))$ and the lump-sum tax, the representative

take into account that an optimal standard-debt-contract (SDC) requires the borrowers be risk neutral (Gale and Hellwig 1985) In addition, if the producer of consumption goods were also the borrower and subject to both the aggregate and idiosyncratic shocks, two problems may arise: (i) Since households provide labor services and get paid the same wage, then the question would be why both the households and financial intermediaries cannot infer what the output and then the actual shocks are from the wage rate and the return to capital; (ii) If the consumption firm had a very low shock and is unable to pay its workers, some households would end up without income to consume.

young agent solves the following problem

$$\max_{\{c_{1,t},c_{2,t+1},d_t\}} \quad \left[log\left(c_{1,t}\right)+\beta log\left(c_{2,t+1}\right)\right], \tag{3a}$$

$$\text{s.t.} \quad c_{1,t}+d_t \leq w(k_t)-\tau_t, \tag{3b}$$

$$c_{2,t+1} \leq d_t^s r^d(k_{t+1}), \tag{3c}$$

$$c_{1,t} \geq 0, \quad c_{2,t+1} \geq 0, \quad d_t \geq 0. \tag{3d}$$

After substituting $c_{1,t}$ and $c_{2,t+1}$ and solving for the first-order conditions (FOCs), the optimal choice and supply of deposits is

$$d^s(k_t) = \beta\frac{w(k_t)-\tau_t}{(1+\beta)}. \tag{4}$$

In this environment, the income and substitution effects cancel out, while the stochastic discount factor (SDF), or intertemporal marginal rate of substitution (IMRS), is given by

$$m_{t+1} = \beta\frac{c_{1,t}}{c_{2,t+1}} = \frac{1}{r^d(k_{t+1})} = m(k_{t+1}). \tag{5}$$

The initial old generation is endowed with an amount of consumption goods a^0 that is consumed ($c_{2,1}=a^0$).

2.3 Financial Intermediary

At the beginning of each period the financial intermediaries receive deposits from young households $d^s(k_t)$, which are taken as equity shares, and decide how much to invest in a one-period risk-free government bond (b_t^d), which pays a risk-free gross return R_{t+1}^f, and to lend to the young capital producers (l_t). Define $r^l : \mathbb{R}^+ \to \mathbb{R}^+$ as the gross return to lending that is a function of the capital stock. Before defining the problem faced by a typical financial intermediary, the following assumption ensures a minimum demand of government bonds:[10]

Assumption 1. *The financial intermediary is required to hold a fraction $\kappa \in (0,1)$ of its deposits as minimum reserves, $RR_t = \kappa d(k_t)$, which are invested in the risk-free government bonds.*

Therefore, the total demand of government bonds is the sum of the required minimum reserves over deposits and the excess demand, $B_t^d = RR_t + b_t^d$.

[10]This is useful in case of a corner solution. The reserve requirement can also introduce an additional policy parameter (κ).

Let define the net cash-flow in period t:

$$W_t = B_{t-1}R_t^f + l_{t-1}r^l(k_t) - d_{t-1}r^d(k_t) + d_t^d - l_t^s - B_t^d \geq 0, \tag{6}$$

where the nonnegative condition is imposed to guarantee solvency.

Each typical financial intermediary maximizes the discounted present value of the net cash flow, taking as given prices $(R_{t+1}^f, r^l(k_{t+1}), r^d(k_{t+1}))$ and the deterministic SDF[11]

$$\max_{\{d_t^d, b_t^d, l_t^s\}} \quad [W_t + m(k_{t+1})W_{t+1}], \tag{7a}$$

$$\text{s.t.} \quad B_t^d \geq \kappa d_t^d, \tag{7b}$$

$$k_{t+1} = f(l_t) \text{ , the law of motion of capital,} \tag{7c}$$

and subject to the solvency condition, with $B_t^d = \kappa . d_t^d + b_t^d$. Let μ_t be the Lagrange multiplier for constraint (7b). The FOCs are:

$$d_t^d : \qquad 1 - m(k_{t+1})r^d(k_{t+1}) - \kappa\mu_t = 0, \tag{8a}$$

$$b_t^d : \qquad -1 + m(k_{t+1})R_{t+1}^f + \mu_t = 0, \tag{8b}$$

$$l_t : \qquad -1 + m(k_{t+1})r^l(k_{t+1}) = 0. \tag{8c}$$

Two cases can be analyzed:

(i) A positive excess demand of goverment bond ($b_t^d > 0$) implies $\mu_t = 0$ and $B_t^d > \kappa . d_t^d$. In this case, from combining the FOCs we get the equality of returns:

$$R_{t+1}^f = r^d(k_{t+1}) = r^l(k_{t+1}). \tag{9}$$

(ii) If (7b) holds with equality, then $\mu_t > 0$ and $b_t^d = 0$. From (8a) and (8b) we get the relationship between the return on deposits and the risk-free rate:

$$\mu_t(1-\kappa) = \left[r^d(k_{t+1}) - R_{t+1}^f\right]. \tag{10}$$

[11] When the free-entry condition is imposed, the financial intermediary problem would be equivalent to maximize the household utility subject to the financial constraint $W_t + m(k_{t+1})W_{t+1} = 0$. Although this is a common practice, as is discussed in the second part, in this model the intermediary will be able to diversify the capital-producer risk and cover itself from the aggregate volatility. Therefore, it is not reasonable to assume that the financial intermediary will behave as a risk-averse agent.

Since left-hand side and the SDF are strictly positive, it must be the case that $r^d(k_{t+1}) > R^f_{t+1}$. But from (8c) and (5) $r^l(k_{t+1}) = r^d(k_{t+1})$. Therefore, in the corner solution, the return to lending must be greater than the risk-free return:

$$r^d(k_{t+1}) = r^l(k_{t+1}) > R^f_{t+1}. \tag{11}$$

This is an intuitive result, that is, if the return from lending is always greater than the return of government bonds, the financial intermediary only keeps the required reserves in bonds ($B^d_t = \kappa d^d_t$), lending the remaining funds to capital-producers, $l^s_t = (1-\kappa)d^d_t$.

However, only the first case will survive in equilibrium. Since $m(k_{t+1})$ is the inverse of $r^d(k_{t+1})$, FOC (8a) implies that μ_t must be zero. Additionally, the second case would generate a loss equivalent to $\kappa d_t(R^f_{t+1} - r^d(k_{t+1}))$, thus, it is ruled out. Therefore, only (9) will be considered in equilibrium.

2.4 Capital Producers

Each period, a *unit mass* of two-period lived capital producers has access to the technology for producing capital goods. They only care about the second-period consumption. As in Labadie (1998) and Gale and Hellwig (1985), since all producers are ex-ante identical and there is a rationing in the size of loans, in period one they demand the same amount of funds (l_t) to invest and produce capital goods, which are available to rent next period. The production is subject to the producer-specific shock θ_i (idiosyncratic shock) that is only known by each producer. The technology $\theta_i l^\alpha_t$ defines the individual production of capital goods, which is rented to the producers of consumption goods, receiving a payment of $\rho(k_{t+1})\theta_i l^\alpha_t$. The following assumption on the idiosyncratic shock is based on the simplified environment proposed by Bernanke et al. (1999).

Assumption 2. *Let the producer-specific shock* (θ_i) *be an i.i.d random variable with a continuous twice-differentiable cumulative distribution function* $G(\tilde{\theta}) = prob(\theta \leq \tilde{\theta})$ *over a non-negative support* $\left[\underline{\theta}, \bar{\theta}\right]$, *with a probability density function* $g(\tilde{\theta}) = \dfrac{dG}{d\tilde{\theta}}$, *and* $E(\theta_i) = 1$.

Although the financial intermediary does not know the realization of each producer's shock in $t+1$, they have the technology to monitor at any time, optimally used only when the capital producer claims bankruptcy. Monitoring is assumed to be deterministic, a capital producer announcing insolvency will be monitored with certainty.[12]

To observe the actual return, the financial intermediary must pay a fraction $\gamma \in (0,1)$ of the producer's return in the second period, $\gamma\rho(k_{t+1})\theta_i l_t^\alpha$. When $\rho(k_{t+1})\theta_i l_t^\alpha > l_t r^l(k_{t+1})$, the producer might falsely claim bankruptcy, however, he could consume at most $(1-\gamma)\,\rho(k_{t+1})\theta_i l_t^\alpha - l_t r^l(k_{t+1})$, which is always lower than the consumption from telling the truth: $\rho(k_{t+1})\theta_i l_t^\alpha - l_t r^l(k_{t+1}) \geq 0$. Therefore, there is no need of an incentive-compatibility constraint.

As all capital producers are ex-ante identical and receive the same credit from the financial intermediary, the subscript i can be dropped to work with the representative capital producer. The break-even producer's shock $(\hat{\theta})$ is defined when the rental revenue only covers the lending cost:

$$\rho(k_{t+1})\hat{\theta} l_t^\alpha = l_t r^l(k_{t+1}). \tag{12}$$

This threshold characterizes the payment scheme: the producer announces default when $\theta < \hat{\theta}$, or repays the loan when $\theta \geq \hat{\theta}$. Given the realization of the idiosyncratic shock, the financial intermediary receives the following payoff:

$$\phi(k_{t+1}, l_t, \gamma, \theta, \hat{\theta}) = \begin{cases} (1-\gamma)\,\rho(k_{t+1})\theta l_t^\alpha & \text{if } \theta < \hat{\theta} \\ \rho(k_{t+1})\hat{\theta} l_t^\alpha & \text{if } \theta \geq \hat{\theta}. \end{cases} \tag{13}$$

Therefore, the producer's payoff is

$$\omega(k_{t+1}, l_t, \theta, \hat{\theta}) = \begin{cases} 0 & \text{if } \theta < \hat{\theta} \\ (\theta - \hat{\theta})\rho(k_{t+1}) l_t^\alpha & \text{if } \theta \geq \hat{\theta}. \end{cases} \tag{14}$$

When the capital producer is in default, all its revenues (net of monitoring costs) are paid to the lender, leaving nothing to consume, and the economy suffers a loss of $\gamma\rho(k_{t+1})\theta l_t^\alpha$. Taking into account the *optimal contract* defined in (13), the capital-producer's expected

[12] Stochastic monitoring is studied by Bernanke and Gertler (1989)

payment (Φ) is

$$\Phi(k_{t+1}, l_t, \gamma, \hat{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(k_{t+1}, l_t, \gamma, \theta, \hat{\theta}) dG = \rho(k_{t+1}) l_t^{\alpha} \left[\int_{\underline{\theta}}^{\hat{\theta}} (1-\gamma)\, \theta dG + \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG \right]. \quad (15)$$

Following Bernanke, Gertler, and Gilchrist (1999), I define the expression in brackets on the right hand side as the expected net-payment relative to the expected revenue, which can be divided into two different terms:

(i) the expected gross payment

$$\Lambda(\hat{\theta}) \equiv \int_{\underline{\theta}}^{\hat{\theta}} \theta dG + \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG,$$

which is strictly increasing and concave since $\Lambda'(\hat{\theta}) = 1 - G(\hat{\theta}) > 0$ and $\Lambda''(\hat{\theta}) = -g(\hat{\theta}) < 0$.

(ii) and the expected monitoring cost

$$\gamma\Omega(\hat{\theta}) \equiv \gamma \int_{\underline{\theta}}^{\hat{\theta}} \theta dG,$$

which is strictly increasing and convex, assuming $\hat{\theta} g(\hat{\theta})$ is increasing, a property that is satisfied for most common distributions (for example the uniform distribution and any monotonically increasing transformation of a normal distribution). Thus, $\gamma\Omega'(\hat{\theta}) = \gamma\hat{\theta} g(\hat{\theta}) > 0$ and $\gamma\Omega''(\hat{\theta}) = \gamma \left[g(\hat{\theta}) + \hat{\theta} g'(\hat{\theta}) \right] > 0$.

Using this notation, the expected payment (15) can be rewritten as

$$\Phi(k_{t+1}, l_t, \gamma, \hat{\theta}) = \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) \right]. \quad (16)$$

Given the properties of $\Lambda(\hat{\theta})$ and $\Omega(\hat{\theta})$, the producer's expected payment is a nonmonotonic function with respect to the cutoff point $\hat{\theta}$

$$\frac{d\Phi(\cdot)}{d\hat{\theta}} = \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta}) \right] \gtreqless 0, \quad (17)$$

reaching a global maximum when $\Lambda'(\theta^m) = \gamma\Omega'(\theta^m)$. The expected payment is increasing on the interval $(\underline{\theta}, \theta^m)$ and decreasing after $(\theta^m, \bar{\theta})$.

Since there is a unit mass of capital producers, as is pointed out by Uhlig (1996), integration over individual producers is the same as integrating over all possible realizations of θ. Let capital producers be indexed by $j \in [0, 1]$, and their shocks ($\theta(j)$) be identically distributed and pairwise uncorrelated with a common mean $E[\theta] = 1$ (Assumption 2), then

$$\int_0^1 \theta(j)dj = 1. \tag{18}$$

As a consequence, from the perspective of each financial intermediary, the idiosyncratic risk is eliminated by the expected payment defined in (16).

The Optimal Choice of Investment. The capital producer chooses the cutoff point $\hat{\theta}$ and the investment level l_t, taking as given prices $(R^f_{t+1}, r^l(k_{t+1}), r^d(k_{t+1}))$ and the expected payoff that results from the optimal contract (13)-(14). To simplify steps, the repayment amount $r^l(k_{t+1})\, l_t$ is equivalent to $R^f_{t+1} l_t$, given the equality of returns in equilibrium. From this perspective, each financial intermediary must receive at least the opportunity cost of lending in expectation. Let's define the set of feasible loanable funds: $L(k_t) = \{l \in y(k_t) : 0 < l \leq (1-\kappa)d(k_t)\}$. Thus, the capital-producer's decision problem is as follows

$$\max_{\hat{\theta}, l_t \in L(k_t)} \left\{ \max \left\{ \left[E(\theta) - \Lambda(\hat{\theta}) \right] \rho(k_{t+1}) l_t^{\alpha}, 0 \right\} \right\}, \tag{19a}$$

$$\text{s.t.} \quad \rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) \right] \geq R^f_{t+1} l_t, \tag{19b}$$

$$k_{t+1} = f(l_t), \text{ the law of motion of capital.} \tag{19c}$$

Let λ be the Lagrange multiplier for constraint (19b). The Lagrangian function for a non-zero expected payoff is

$$\mathcal{L} = \left[1 - \Lambda(\hat{\theta}) \right] \rho(k_{t+1}) l_t^{\alpha} + \lambda \left[\rho(k_{t+1}) l_t^{\alpha} \left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) \right] - R^f_{t+1} l_t \right]. \tag{20}$$

Note that $E(\theta) = 1$ by Assumption (2), so $0 < 1 - \Lambda(\hat{\theta}) < 1$.[13] The case when $\lambda = 0$ is ruled out because the optimizing capital-producer will never pay more than the repayment agreement, thus constraint (19b) holds with equality.

The FOCs for an *interior solution* ($\lambda > 0$) are

$$\hat{\theta}: \qquad \lambda(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})}, \tag{21a}$$

$$l_t: \qquad \alpha\rho(k_{t+1})l_t^{\alpha-1} = \frac{\lambda(\hat{\theta})R^f_{t+1}}{\left[1 - \Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{21b}$$

$$\lambda: \qquad \rho(k_{t+1})l_t^{\alpha}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right] = R^f_{t+1}l_t. \tag{21c}$$

The second-order condition (SOC):

$$[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})]\lambda - \Lambda''(\hat{\theta}) < 0,$$

is always satisfied, since $\Lambda'' < 0$, $\gamma\Omega'' > 0$, and $\lambda > 1$ (see Appendix B).

To analyze possible interior solutions, let $\varepsilon(\hat{\theta})$ be the *excess return* from lending to the capital producer, from the FOC (21b): [14]

$$\varepsilon(\hat{\theta}) = \frac{\lambda(\hat{\theta})}{\left[1 - \Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{22}$$

and using (21a) to substitute $\lambda(\hat{\theta})$ we get

$$\varepsilon(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]}, \tag{23}$$

which is greater than 1 and increasing in $\hat{\theta}$ (see Appendix C).

Proposition 2.1. *Let $\hat{\theta}^*$ and l_t^* be the optimal cutoff point and investment respectively that solve the problem (19a) subject to (19b)-(19c). To get an interior solution, $\hat{\theta}^*$ must be chosen on the increasing part of the expected return function (16), this is when $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$.*

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[13] As a result, $\Lambda(\hat{\theta}) = \int_{\underline{\theta}}^{\hat{\theta}} \theta dG - \int_{\hat{\theta}}^{\bar{\theta}} \hat{\theta} dG < 1$.

[14] When aggregate risk is introduced in the next section, the excess return will represent the risk premium.

Then, both $\hat{\theta}^*$ and l_t^* are unique, defining the optimal demand of government bonds by the financial intermediary.

Proof. For the first part, as $\hat{\theta}^*$ gets closer to the maximizer of the expected payment (θ^m), where $\Lambda'(\theta^m) = \gamma\Omega'(\theta^m)$, the $\lim_{\hat{\theta}\to\theta^m} \lambda(\hat{\theta}) = +\infty$. This implies that

$$\lim_{\hat{\theta}\to\theta^m} \frac{\lambda(\hat{\theta})}{\left[1-\Lambda(\hat{\theta})\right] + \lambda(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} = \frac{1}{\Lambda(\theta^m) - \gamma\Omega(\theta^m)}. \tag{24}$$

Substitute (24) in (21b) to get

$$\rho(k_{t+1})l_t^{\alpha-1}\left[\Lambda(\theta^m) - \gamma\Omega(\theta^m)\right] = \frac{R_{t+1}^f}{\alpha}. \tag{25}$$

Then, it is possible to show that if (25) holds, then (21c) will never hold with equality and the expected return of lending is always greater than its opportunity cost for any value of l:

$$\frac{R_{t+1}^f}{\alpha} > R_{t+1}^f, \quad \text{or} \quad R_{t+1}^f(1-\alpha) > 0. \tag{26}$$

In this case the financial intermediary has an incentive to lend out all the available resources (net of the reserve requirement) to capital producers, reaching the upper bound of the set of feasible loanable funds: $l_t^* = (1-\kappa)\, d(k_t)$. If $\hat{\theta}^*$ is chosen in the decreasing part of the expected return function (16), when the cutoff $\hat{\theta} \in (\theta^m, \bar{\theta})$, then $\lambda(\hat{\theta})$ would be negative.

Once the interior solution is guaranteed by $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$, the optimal values of both the break-even point and investment are the solution to the system of equations formed by the FOCs (21a)-(21c). The equilibrium break-even point ($\hat{\theta}^*$) is obtained by substituting (21c) in (21b) and using the excess return defined in (23) to get

$$\frac{\Lambda'(\hat{\theta}^*)}{\left[\Lambda'(\hat{\theta}^*) - \gamma\Omega'(\hat{\theta}^*)\right]\left[1-\Lambda(\hat{\theta}^*)\right] + \Lambda'(\hat{\theta}^*)\left[\Lambda(\hat{\theta}^*) - \gamma\Omega(\hat{\theta}^*)\right]} = \frac{\alpha}{\Lambda(\hat{\theta}^*) - \gamma\Omega(\hat{\theta}^*)}. \tag{27}$$

The right hand side of (27) is the gross-return on investment (GRI), that is, the ratio of the expected marginal revenue of capital producer (EMR) to return of loans ($r^l(k_{t+1})$):

$$
\begin{aligned}
GRI(\hat{\theta}) &= \frac{EMR(k_{t+1}, l_t)}{r^l(k_{t+1})} \\
&= \frac{\alpha\rho(k_{t+1})l_t^{\alpha-1}}{\rho(k_{t+1})l_t^{\alpha-1}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} \\
&= \frac{\alpha}{\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})}.
\end{aligned}
$$

Since GRI is strictly decreasing in $\hat{\theta}$ and the excess return is strictly increasing in $\hat{\theta}$, there exits a unique solution $\hat{\theta}^*$ to (27). All capital-producers face the same distribution of indiosyncratic risk, thus $\hat{\theta}_i^* = \hat{\theta}^*$ for all i.

Given the optimal choice $\hat{\theta}^*$, the optimal level of investment (l_t^*) is the solution of the FOC (21b),

$$
\alpha\rho(k_{t+1})(l_t^*)^{\alpha-1} = \varepsilon(\hat{\theta}^*)R_{t+1}^f. \tag{28}
$$

Again, the left-hand side of (28) is the expected marginal return of capital producer ($EMR(k_{t+1}, l_t)$) that has the following properties:

(i) $\lim_{l\to 0} EMR(k_{t+1}, l_t) = +\infty$.

(ii) The lowest value is given by the corner solution: $EMR(k_{t+1}, (1-\kappa)d_t) = \dfrac{\alpha\rho(k_{t+1})}{[(1-\kappa)d_t]^{1-\alpha}}$.

(iii) It is decreasing in l_t as $\dfrac{\partial EMR(\cdot)}{\partial l_t} = \alpha(\alpha-1)\rho(k_{t+1})l_t^{\alpha-2} < 0$.

(iv) And convex since $\dfrac{\partial^2 EMR(\cdot)}{\partial l_t^2} = \alpha(\alpha-1)(\alpha-2)\rho(k_{t+1})l_t^{\alpha-3} > 0$.

By solving (28), any capital producer i equalizes his EMR_i, which is convex and strictly decreasing in l, with $\varepsilon(\hat{\theta}^*)R_{t+1}^f$, which does not depend on l. Then, the capital-producer i chooses a unique optimal level of investment given by

$$
l_i^*(\varepsilon(\hat{\theta}^*), R_{t+1}^f, \rho(k_{t+1})) = \left[\frac{\alpha\rho(k_{t+1})}{\varepsilon(\hat{\theta}^*)R_{t+1}^f}\right]^{\frac{1}{1-\alpha}}. \tag{29}
$$

□

2.5 Government

At the beginning of each period, the government announces the risk-free rate (R^f_{t+1}) at which will offer the one-period bonds and choses the lump-sum taxes (τ_t). Once these policy variables are chosen, the total government spending per worker (g_t) is endogenously determined by the total demand of government bonds (B^d_t), net of previous debt repayment, to satisfy

$$g_t = \tau_t + B^d_t - R^f_t B_{t-1}, \tag{30}$$

$$\lim_{t\to\infty} \frac{B_t}{{R^f}^{t-1}} = 0 \text{ (no-Ponzi constraint)}, \tag{31}$$

$$\text{and } B_0 \text{ is given.}$$

In other words, the government issues bonds to cover the financial sector's demand by assuming a perfectly elastic supply of bonds at the announced rate. The risk-free rate (R^f_{t+1}) can be thought as a policy rate that affects the entire economy through (28).[15] In addition, the reserve requirement (κ) could also be used as another policy instrument to reduce the loanable funds available to the private sector and increase the total demand of goverment bonds.

2.6 Equilibrium and the Financial Depth Indicator

In this section, I examine the steady state taking as given the risk-free return on government bonds ($R^f_{t+1} = R^f$). Any change in R^f will result in a transition to a new steady state.

The law of motion of capital stock for a given k_0 and R^f is defined by

$$k(l^*, E(\theta)) = E(\theta){l^*}^{\alpha} \tag{32}$$

$$= {l^*}^{\alpha} \quad \text{(by Assumption 2).} \tag{33}$$

[15] In the last section, I propose an extension of this model, where the government can choose the level of spending but without the perfectly-elastic supply of bonds.

Then, the total output per worker in steady state is

$$y(l^*, A) = A{l^*}^{\alpha\delta}, \tag{34}$$

and the factor payments are

$$w(l^*, A) = (1-\delta)A{l^*}^{\alpha\delta}, \tag{35}$$

$$\rho(l^*, A) = \delta A{l^*}^{\alpha(\delta-1)}. \tag{36}$$

From (28) and (36), the steady state level of investment is defined as follows

$$l^*(\varepsilon(\hat{\theta}^*), R^f, A) = \left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*)R^f}\right]^{\frac{1}{1-\alpha\delta}}, \tag{37}$$

which is decreasing in both the excess return and the risk-free return, but increasing in the total factor productivity.[16]

The steady state level of deposits (d^*) is given by $d^s = \beta\dfrac{w(l^*, A) - \tau_t}{(1+\beta)} = d^d = d^*$. At the given R^f, the government supplies bonds to absorb its demand in steady state ($B^{d^*} = d^* - l^*$), validating a spending level g^* that satisfies (30).

The resource constraint in the economy is

$$y(l^*, A) = c_1^* + c_2^* + g^* + l^* + \int_{\underline{\theta}}^{\bar{\theta}} \omega(k, l, \theta, \hat{\theta})g(\theta)d\theta \ + DWL, \tag{38}$$

[16]Taking the partial derivatives:

$$\frac{\partial l^*}{\partial\varepsilon(\hat{\theta}^*)} = -\frac{1}{\varepsilon(\hat{\theta}^*)}\left(\frac{1}{1-\alpha\delta}\right)\left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*)R^f}\right]^{\frac{1}{1-\alpha\delta}} < 0,$$

$$\frac{\partial l^*}{\partial R^f} = -\frac{1}{R^f}\left(\frac{1}{1-\alpha\delta}\right)\left[\frac{\alpha\delta A}{\varepsilon(\hat{\theta}^*)R^f}\right]^{\frac{1}{1-\alpha\delta}} < 0,$$

$$\frac{\partial l^*}{\partial A} = A^{\frac{\alpha\delta}{1-\alpha\delta}}\left(\frac{1}{1-\alpha\delta}\right)\left[\frac{\alpha\delta}{\varepsilon(\hat{\theta}^*)R^f}\right]^{\frac{1}{1-\alpha\delta}} > 0.$$

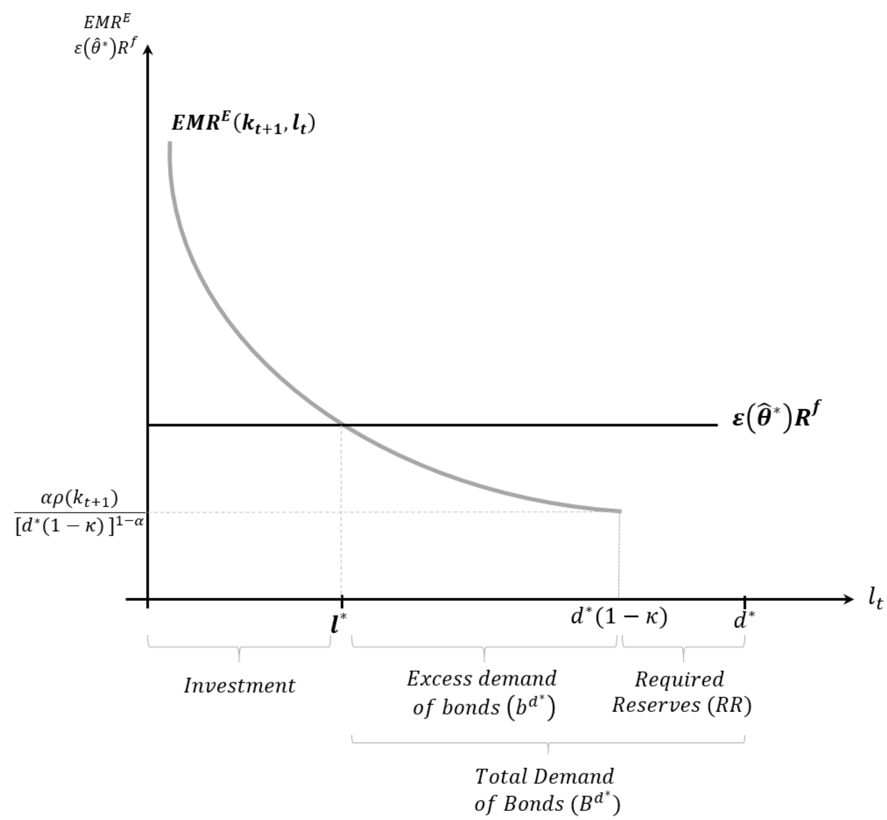
where DWL is the deadweight loss from monitoring, which is defined as follows

$$DWL = \rho(l^*, A){l^*}^{\alpha}\gamma\Omega(\hat{\theta}) \tag{39}$$

$$= \delta y(l^*, A)\gamma\Omega(\hat{\theta}). \tag{40}$$

To get a graphical representation of the optimal decision and steady- state values in the entire economy, let EMR^E be aggregate expected marginal-return function. Since all capital producers are ex-ante identical, we get $EMR^E = \alpha\rho(k_{t+1})l_t^{\alpha-1}$, which is a convex and strictly decreasing function on l_t. Given R^f and $k_0 > 0$, Figure 1 depicts the optimal decision (28) for the economy, resulting in a unique steady-state level of investment and demand of government bonds.

Figure 1: Aggregate Investment and Demand of Government Bonds in Steady State



The effect of changes in R^f, the excess return, and productivity (A) are graphically analyzed in Appendix A. For instance, if the government announces an increase in the risk-free rate from R^f to $R^{f'}$ to finance more spending (g). Everything else equal, this policy would generate a reduction in the private investment, a phenomenon known as *crowding out effect*. In this framework, the increase of government expenditures is equivalent to the reduction in private investment as is depicted in Figure A1.

Finally, if the financial depth (FD) is proxied by the level of private credit extended by the financial intermediaries as a share of total output, the capital producers are the only private agents who borrows from the financial intermediary.[17] Therefore, given R^f and k_0, the financial depth indicator in steady state (FD^*) is

$$\begin{aligned} FD^*(\varepsilon(\hat{\theta}^*), R^f) &= \frac{l^*(\varepsilon(\hat{\theta}^*), R^f, A)}{y(l^*, A)} \\ &= \frac{{l^*}^{1-\alpha\delta}}{A} \\ &= \frac{\alpha\delta}{\varepsilon(\hat{\theta}^*)R^f} \quad \text{by (37).} \end{aligned} \tag{41}$$

The degree of financial deepening in equilibrium is decreasing in both the excess return and the risk-free return. However, unlike the investment level, FD^* does not depend on the level of total factor productivity.

3 Introducing Aggregate Risk into the Benchmark Model

The preceding section provides a benchmark model with equilibrium levels of investment and financial depth in an economy without aggregate risk, and only subject to the idiosyncratic risk in the production of capital goods. In this section, an aggregate productivity shock (S_t) is introduced that will affect both the TFP (A_t) and the distribution of the producer-specific shock (G). For simplicity, there are two possible states of the world with equal probability, low ($A(L)$) and high ($A(H)$) productivity with $\pi(L) = \pi(H) = 0.5$, and let σ_p be the standard deviation of productivity shocks. Hereafter the subscripts "V" and "v" will identify the functions and chosen variables, respectively, associated with the stochastic environment.

Some empirical studies find that permanent changes in macroeconomic risk can explain

[17] This might be a limitation of the model, since the total credit to the private sector typically includes a variety of loans to households, financial institutions, and funds used as working capital. From the national account perspective, the FD indicator in this model would be equivalent to the country's investment rate.

structural changes in the equity price trends and risk premium.[18] In line with this literature, I propose a link between the TFP volatility and the distribution of idiosyncratic risk, which will have consequences on the risk premium. The following assumption summarizes this key link between the macroeconomic risk and individual risk:

Assumption 3. *A change in the aggregate risk or TFP volatility generates a mean-preserving spread (MPS) change in the original distribution of the idiosyncratic shock over the same non-negative support* $\left[\underline{\theta}, \bar{\theta}\right]$*. The standard deviation of the new distribution depends on the volatility of the productivity shock (σ_p), that is, the more volatile is the economy, the riskier is the production of capital goods.*

These properties are formalized next.

Definition 3.1. *Let $\theta_{vi} \sim G_V$ and $\theta_i \sim G$ be random variables associated with the realizations of the producer-specific shock for an economy with and without aggregate volatility respectively. G_V is a mean-preserving spread G if and only if*

$$\theta_{vi} \stackrel{d}{=} \theta_i + z,$$

for some noise z such that $E(z|\theta_i) = 0 \ \forall \theta_i$. In addition, the variance of the noise z is assumed to be an increasing function of the aggregate volatility: $\partial Var(z)/\partial \sigma_p > 0$.

Since the MPS is a special case of the Second Order Stochastic Dominance (SOSD), G second-order stochastically dominates G_V, therefore the following conditions hold[19]:

(i) The two distributions have the same mean:

$$\int_{\underline{\theta}}^{\bar{\theta}} \theta dG_V = \int_{\underline{\theta}}^{\bar{\theta}} \theta dG. \tag{42}$$

(ii) Integrating over the difference in the distributions is a non-negative function of the

[18] By using postwar data, Lettau et al. (2007) show that there is a strong correlation between the macroeconomic volatility and asset prices, so with the equity risk premium as well, not only in United States but also at international level. Bansal et al. (2005) also find that GARCH volatility measures can explain quarterly price-dividend ratios in United Sates, United Kingdom, Germany, and Japan.

[19] For discussions and economic applications of the mean-preserving increase in risk see Rothschild and Stiglitz (1970, 1972) and Diamond and Stiglitz (1974)

aggregate volatility:

$$\Gamma(t, \sigma_p) = \int_{\underline{\theta}}^{t} \left[G_V(\theta, \sigma_p) - G(\theta) \right] d\theta \geq 0, \quad \text{for all} \quad \underline{\theta} \leq t \leq \bar{\theta}. \tag{43}$$

From Definition 3.1 and Assumption 3, an increase in the aggregate risk will generate a mean preserving increase in the capital-producer's specific risk. Thus, it is reasonable to impose the following working assumption:[20]

Assumption 4. *A mean-preserving spread increase in the idiosyncratic risk will raise the default probability for a given cut off point: $G_V(\hat{\theta}_v, \sigma_p') > G_V(\hat{\theta}_v, \sigma_p)$ when $\sigma_p' > \sigma_p$. By moving weight from the center of the probability density function to the tails, without affecting the mean, the pdf is assumed to shift up over the interval $[\underline{\theta}, \hat{\theta}_v]$.*

By introducing aggregate volatility, the factor payments depend on the current state,

$$w(k_t, S_t) = (1 - \delta)\, A(S_t) k_t^{\delta}, \tag{44}$$

$$\rho(k_t, S_t) = \delta A(S_t) k_t^{(\delta - 1)}, \tag{45}$$

as well as the optimal supply of deposits:

$$d_{v,t}^{s} = \beta \frac{w(k_t, S_t) - \tau_t}{(1 + \beta)}. \tag{46}$$

As in previous section, the financial intermediary maximizes the discounted present value of the net cash flow, but now the next period cash flow (W_{t+1}) and the SDF depend on the next period aggregate shock (S_{t+1}). Then, FOCs for the stochastic version of (7) are

$$d_{v,t}^{d}: \qquad 1 - E_t \left[m(k_{t+1}, S_{t+1}) r^d(k_{t+1}, S_{t+1}) \right] - \kappa \mu_{v,t} = 0, \tag{47a}$$

$$b_{v,t}^{d}: \qquad -1 + E_t \left[m(k_{t+1}, S_{t+1}) R_{t+1}^{f} \right] + \mu_{v,t} = 0, \tag{47b}$$

$$l_{v,t}: \qquad -1 + E_t \left[m(k_{t+1}, S_{t+1}) r^l(k_{t+1}, S_{t+1}) \right] = 0, \tag{47c}$$

[20] The assumption of equal means requires that the distribution functions G and G_V have to cross at least once, therefore, there exits the possibility that $G_V(\hat{\theta}, \sigma_p) \leq G(\hat{\theta})$. However, that situation should be ruled out, because it would result in a counterintuitive prediction, that is, a relatively lower default probability when the aggregate volatility increases.

where $\mu_{v,t}$ is the Lagrangian multiplier for the required-reserve constraint.

For the same reason as explained in the deterministic benchmark, only the case when $\mu_{v,t} = 0$ and $b^d_{v,t} > 0$ is considered. Combining the FOCs and defining the $m(k_{t+1}, S_{t+1}) = 1/r^d(k_{t+1}, S_{t+1})$, we get the standard relationships

$$E_t\left[\frac{r^l(k_{t+1}, S_{t+1})}{r^d(k_{t+1}, S_{t+1})}\right] = E_t\left[\frac{1}{r^d(k_{t+1}, S_{t+1})}\right] R^f_{t+1} = 1. \tag{48}$$

3.1 The Capital-Producer Problem with Idiosyncratic and Aggregate Risk

When aggregate risk is introduced, the optimal decision of capital producer faces two major changes. First, there is a riskier distribution for his specific shock that increases the default probability at a given break-even point. Second, his income from renting the new capital is subject to the next period state, i.e. $\rho(k_{t+1}, S_{t+1})\theta_v l^\alpha_{v,t}$.

Since the standard debt contract (SDC) is characterized by a fixed repayment agreement (Gale and Hellwig, 1985), let $\hat{r}$ be the fixed payment per unit of loan. Thus, the capital producer is breakeven when

$$\rho(k_{t+1}, S_{t+1})\hat{\theta}_v l^\alpha_{v,t} = \hat{r} l_{v,t}, \tag{49}$$

where $\hat{\theta}_v$ is the break-even point for the stochastic problem. Given $(k_{t+1}, S_{t+1}, l_{v,t})$, the financial intermediary receives the following payoff:

$$\phi_V(k_{t+1}, S_{t+1}, l_{v,t}, \gamma, \theta_v, \hat{\theta}_v) = \begin{cases} (1-\gamma)\,\rho(k_{t+1}, S_{t+1})\theta_v l^\alpha_{v,t} & \text{if } \theta_v < \hat{\theta}_v \\ \hat{r} l_{v,t} & \text{if } \theta_v \geq \hat{\theta}_v. \end{cases} \tag{50}$$

Using (49) to substitute $\hat{r} l_{v,t}$, the producer's expected payment from the benchmark model (16) is replaced by:

$$\Phi_V(k_{t+1}, S_{t+1}, l_{v,t}, \gamma, \hat{\theta}_v, \sigma_p) = \rho(k_{t+1}, S_{t+1}) l^\alpha_{v,t}\left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p)\right], \tag{51}$$

where the expected gross payment and monitoring cost relative to the income are respectively,

$$\Lambda_V(\hat{\theta}_v, \sigma_p) \equiv \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V + \int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V \tag{52}$$

and

$$\gamma\Omega_V(\hat{\theta}_v, \sigma_p) \equiv \gamma \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V. \tag{53}$$

Once the aggregate risk is defined (σ_p), the realization of the aggregate state (S_{t+1}) will affect the return on capital, but not $\left[\Lambda_V(\hat{\theta}, \sigma_p) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p)\right]$.

The expected payment function has the same properties defined in the benchmark case: it is nonmonotonic in the cutoff point, reaching a global maximum at θ_v^m, where it is increasing on the interval $(\underline{\theta}, \theta_v^m)$ and decreasing after.

In sum, the capital producer chooses the break-even point and investment (loans) from the available funds $L(k_t, S_t) = \{l_{v,t} \in y(k_t, S_t) : 0 \leq l_{v,t} \leq (1-\kappa)d(k_t, S_t)\}$, taking as given prices $(R_{t+1}^f, r^l(k_{t+1}, S_{t+1}), r^d(k_{t+1}, S_{t+1}))$, to maximize his expected payoff over the two possible states $S_{t+1} \in \{L, H\}$ with equal probability. Each producer faces the financial intermediary's participation constraint, and the law of motion of capital. The intermediary will voluntarily participate in lending if the expected payment covers at least $r^l(k_{t+1}, S_{t+1})\, l_{v,t}$ or its equivalent in equilibrium $R_{t+1}^f l_{v,t}$. Formally, the representative capital producer solves

$$\max_{\hat{\theta}_v, l_{v,t} \in L(k_t, S_t)} \left\{ \max \left\{ \frac{l_{v,t}^\alpha}{2} \left[E(\theta_v) - \Lambda_V(\hat{\theta}_v, \sigma_p)\right] \left[\rho(k_{t+1}, L) + \rho(k_{t+1}, H)\right], 0 \right\} \right\}, \tag{54a}$$

$$\text{s.t.} \quad \frac{l_{v,t}^\alpha}{2} \left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p)\right] \left[\rho(k_{t+1}, L) + \rho(k_{t+1}, H)\right] \geq R_{t+1}^f l_{v,t}, \tag{54b}$$

$$k_{t+1} = f(l_{v,t}), \text{ the law of motion of capital.} \tag{54c}$$

To simplify notation, hereafter I omit the arguments of functions Λ_V and Ω_V, while the return on capital $\rho(k_{t+1}, S_{t+1})$ is abbreviated as $\rho(S_{t+1})$. Since the distribution of the idiosyncratic shock θ_v is a MPS of that for θ, then $E(\theta_v) = E(\theta) = 1$.

Let λ_v be the Lagrange multiplier for the constraint (54b), the FOCs for an interior solution ($\lambda_v > 0$) are quite similar to those associated with the benchmark model:

$$\hat{\theta}_v: \qquad \lambda_v = \frac{\Lambda'_V}{\Lambda'_V - \gamma\Omega'_V}, \tag{55a}$$

$$l_{v,t}: \qquad \alpha E_t\left[\rho(S_{t+1})\right] l_{v,t}^{\alpha-1} = \frac{\lambda_v R^f_{t+1}}{\left[1-\Lambda_V\right] + \lambda_v\left[\Lambda_V - \gamma\Omega_V\right]}, \tag{55b}$$

$$\lambda_v: \qquad l_{v,t}^{\alpha}\left[\Lambda_V - \gamma\Omega_V\right] E_t\left[\rho(S_{t+1})\right] = R^f_{t+1} l_{v,t}. \tag{55c}$$

The SOC is satisfied when

$$\left[\Lambda''_V - \gamma\Omega''_V\right]\Lambda'_V - \left[\Lambda'_V - \gamma\Omega'_V\right]\Lambda''_V < 0,$$

which is similar to the condition found in the deterministic case, but substituting out λ_v with (55a). As before, an optimizing producer will pay no more than $r^l(k_{t+1}, S_{t+1})\, l_{v,t}$ or, equivalently, $R^f_{t+1} l_{v,t}$, thus, constraint (54b) holds with equality.

In the stochastic framework, the excess return is the *risk premium* from lending to the capital producer, since it does not only depend on the cutoff point, but also on aggregate volatility.

$$\varepsilon_V(\hat{\theta}_v, \sigma_p) = \frac{\lambda_v}{\left[1-\Lambda_V\right] + \lambda_v\left[\Lambda_V - \gamma\Omega_V\right]}. \tag{56}$$

The procedures followed in Preposition (2.1) are still valid in the stochastic environment in order to get the equilibrium break-even point and investment. The former is obtained when the risk premium is equal to the expected gross return on investment ($GRI_V(\hat{\theta}_v, \sigma_p)$). Substituting λ_v with (55a) we obtain:[21]

$$\frac{\Lambda'_V}{\left[1-\Lambda_V\right]\left[\Lambda'_V - \gamma\Omega'_V\right] + \Lambda'_V\left[\Lambda_V - \gamma\Omega_V\right]} = \frac{\alpha}{\Lambda_V - \gamma\Omega_V}. \tag{57}$$

Recall that uniqueness is guaranteed because the left-hand side is increasing, while the

[21]The expected gross return on investment with aggregate risk is

$$GRI_V(\hat{\theta}_v, \sigma_p) = \frac{\alpha l_{v,t}^{\alpha-1} E_t\left[\rho(S_{t+1})\right]}{R^f_{t+1}} = \frac{\alpha l_{v,t}^{\alpha-1} E_t\left[\rho(S_{t+1})\right]}{l_{v,t}^{\alpha-1}\left[\Lambda_V - \gamma\Omega_V\right] E_t\left[\rho(S_{t+1})\right]}, \quad \text{using (55c)}$$
$$= \frac{\alpha}{\Lambda_V - \gamma\Omega_V}.$$

right-hand side is decreasing in the cutoff point ($\hat{\theta}_v$). As was proved for the benchmark case, the optimal cutoff point is chosen in the increasing part of the expected net payment function, i.e. $\hat{\theta}^*_v \in (\underline{\theta}, \theta^m_v)$.

Following the analysis of Section 2.6, we can derive the steady state investment (l^*_v) for given values of the risk-free return ($R^f_{t+1} = R^f$) and initial capital ($k_0 > 0$). After substituting (56) in (55b) at the optimal cutoff point $\hat{\theta}^*_v$ obtained from (57), and replacing $E_t\left[\rho(S_{t+1})\right]$ for its value in equilibrium, we find

$$l^*_v\left(\varepsilon_V(\hat{\theta}^*_v, \sigma_p), R^f, E\left[A(S)\right]\right) = \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}^*_v, \sigma_p)R^f}\right]^{\frac{1}{1-\alpha\delta}}. \tag{58}$$

When the capital producer chooses the amount of borrowing, he considers the expected future return from renting capital. Therefore, the steady state investment in (58) not only depends on the risk premium and the given R^f, but also on the expected value of productivity shocks. As before, l^*_v is decreasing in the risk premium and the return of government bonds, but increasing in $E\left[A(S)\right]$. There is a trade-off between increasing the risk premium and the expected value of the productivity shocks. If there were an increase in both the volatility of productivity shock (σ_p) and its expected value ($E\left[A(S)\right]$), the optimal level of investment (58) would depend of the relative effect of the aggregate shock on the risk premium. This interaction is studied in the following section.

Since the financial intermediary lends out funds to a countably infinite number of risk-neutral producers, the latter bear both the aggregate and individual risks associated with the contract. Even though when the aggregate volatility is in place, the intermediary can diversify the aggregate risk among borrowers.

But what about the output volatility (σ_y)?: After modifying the equilibrium condition for output (34) to account for the aggregate risk, and substituting the steady state investment (58), the output becomes $y(l^*, A(S_t)) = A(S_t){l^*}^{\alpha\delta}$. Therefore, the standard deviation of total output is given by

$$\sigma_y(\sigma_p, l^*_v) = \sigma_p {l^*_v}^{\alpha\delta}, \tag{59}$$

which means that the output volatility is endogenous, and in equilibrium, depends not only on the volatility of the productivity shock, but also on the equilibrium level of investment. Therefore, the following is true:

Result 1. *It follows from (59) and (58) that any change in the aggregate risk (σ_p) has two effects on output volatility (σ_y): a one-to-one direct effect and an indirect one, since the optimal level of investment l_v^* is also a function of σ_p.*

Once the effect of aggregate risk on the optimal level of investment is defined in the next section, Result 1 will be replaced by Result 4.

The steady state level of financial deepening with aggregate risk is

$$FD_v^* \left(\varepsilon_V(\hat{\theta}_v^*, \sigma_p), R^f, E\left[A(S)\right], A(S) \right) = \frac{{l_v^*}^{1-\alpha\delta}}{A(S)} = \left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^*, \sigma_p) R^f} \right] \left[\frac{E\left[A(S)\right]}{A(S)} \right]. \tag{60}$$

The first factor is equivalent to the measure found in benchmark case, while the second one is the direct impact of the aggregate productivity shock. The latter factor represents the first link between the macroeconomic volatility and the financial deepening, which can be summarized as follows

Result 2. *The financial depth indicator in steady state is not only affected by the risk premium and the return of government bond, but it also fluctuates with the current productivity shock.*

This is an intuitive result, since the FD is normalized by the output, thus its dynamic is also a function of the current productivity shock. [22]

Results (1) and (2) reflect the complexity faced by any empirical work that studies relationship between the financial depth (FD) and output volatility (σ_y), because both are endogenous! When the FD is used as the explanatory variable, it is not always easy to find the proper instrumental variable to isolate it from the aggregate risk.

[22] However, if the financial depth is measured in terms of the expected output, the second factor in (60) would be simplified. The resulting indicator would match the form of the FD for the benchmark economy (41), but now with the risk premium being a function of the aggregate risk.

The optimal investment (58) and the financial depth indicator defined in (60) show the importance of having a link between the aggregate risk and the idiosyncratic shock. If this interaction were omitted, the risk premium would not be affected by the aggregate volatility. Then, if there were an increase in both the volatility of the productivity shock (σ_p) and its expected value ($E[A(S)]$), the following results would hold:

(i) From (58), there would be an increase in the level of investment, amplifying the output volatility in (59).

(ii) At the same time, the level of financial deepening (60) would solely depend on the expected productivity shock relative to the actual state.

(iii) An increase in the aggregate risk and its expected value would result in an increase in the optimal level of investment, amplifying the output volatility, and with the possibility of having a deeper financial system.

4 In Search of the Missing Links

In this part, the effect of aggregate risk or TFP volatility is studied in steps. First, the analysis is concentrated on the investment level, or total credit to the private sector, and consequently on the output fluctuations. In a second step, the results are introduced in the financial depth indicator (FD) in order to check the effects on financial deepening.

To isolate the impact of the aggregate risk, let's consider an economy that starts with a certain level of variability on its total factor productivity, say σ_p^B. Now assume a one-time *exogenous* reduction in its fluctuation to σ_p^A, but keeping constant all other features associated with this model, including the mean of productivity shocks, i.e. $E_t\left[A(S_{t+1}^a)\right] = E_t\left[A(S_{t+1}^b)\right] = E_t\left[A(S_{t+1})\right]$ for all periods. If $\sigma_p^A = 0$, the economy is back to the benchmark case. It is also possible to think of this exercise as two economies with different aggregate volatility. Next, this change is assessed in the context of the investment/lending decisions to finally reach some conclusions about its impact on financial depth.

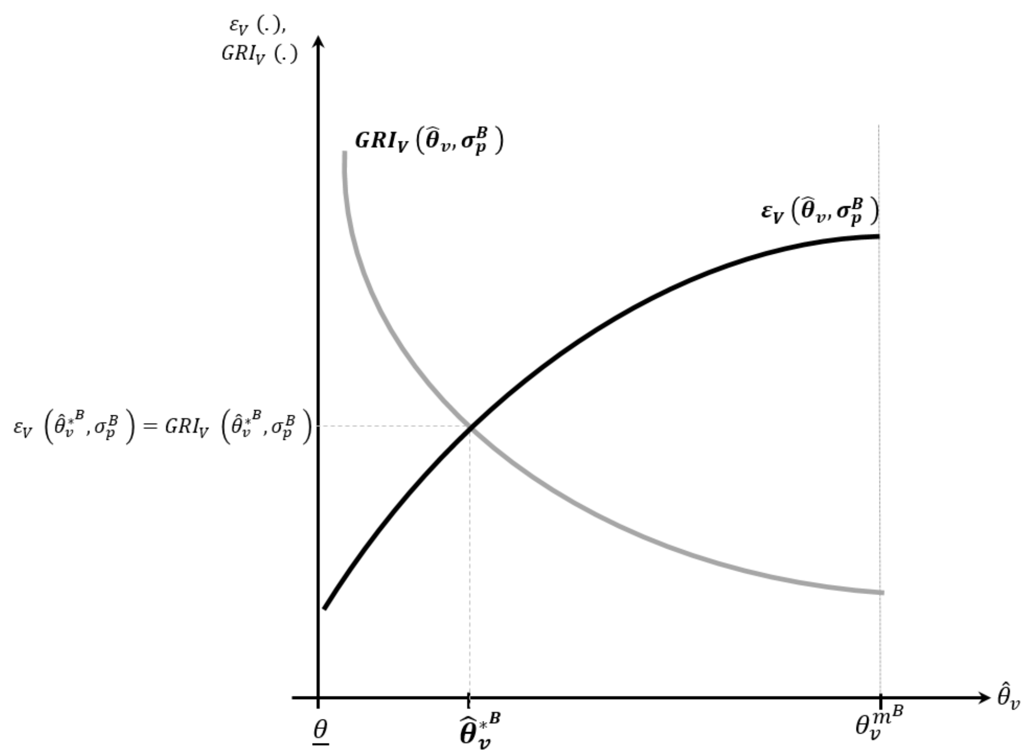
4.1 The Effect of Aggregate Risk on Lending and Output Volatility

Let's start from the optimal decision (57) associated with the initial aggregate risk σ_y^B, but without substituting λ_v from the risk premium for the scope of the analysis:

$$\frac{\lambda_v(\hat{\theta}_v^{*B}, \sigma_p^B)}{\left[1 - \Lambda_V(\hat{\theta}_v^{*B}, \sigma_p^B)\right] + \lambda_v(\hat{\theta}_v^{*B}, \sigma_p^B)\left[\Lambda_V(\hat{\theta}_v^{*B}, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v^{*B}, \sigma_p^B)\right]} = \frac{\alpha}{\Lambda_V(\hat{\theta}_v^{*B}, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v^{*B}, \sigma_p^B)}. \quad (61)$$

To obtain the effect of reducing the aggregate risk on lending, let's analyze how both the risk premium and the expected gross return on investment change. As noted before, the gross return on investment in the right-hand side is decreasing on the break-even point ($\hat{\theta}_v$), while the risk premium (left-hand side) is increasing. Figure 2 depicts these facts, considering that the $GRI(\cdot)$ is a convex function (since $\partial^2 GRI(\cdot)/\partial\hat{\theta}_v^2 > 0$) and assuming a concave $\varepsilon_V(\cdot)$, as long as $\partial^2\varepsilon_V(\cdot)/\partial\hat{\theta}_v^2 < 0$. The convex and concave shapes of these functions will be irrelevant for the final results. At the intersection of the curves, a unique optimal break-even point is found ($\hat{\theta}_v^{*B}$) for the initial situation with σ_p^B.

Figure 2: Risk Premium and the Expected Gross Return on Investment



The following proposition summarizes the individual effect on the GRI function.

Proposition 4.1. *The lower is the aggregate risk (σ_p), the higher is the expected net-payment relative to the expected income in each state,* $\left[\Lambda_V(\hat{\theta}_v, \sigma_p) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p)\right]$*, and the lower is the expected gross return on investment at the cutoff point. Given* $\sigma_p^B > \sigma_p^A$*, this implies that* $GRI_V(\hat{\theta}_v, \sigma_p^B) > GRI_V(\hat{\theta}_v, \sigma_p^A)$*.*

Proof. Let $Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B)$ be the difference in the expected net-payment after reducing the aggregate risk, relative to the expected income in each state, for all possible values of $\hat{\theta}_v$:

$$Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B) = \left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^B)\right] - \left[\Lambda_V(\hat{\theta}_v, \sigma_p^A) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^A)\right]. \tag{62}$$

To simplify the analysis, let's apply integration by parts in (52) and (53), and the fact that $\int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V = \hat{\theta}_v \left(1 - G_V(\hat{\theta}_v)\right)$. Therefore, the gross payment becomes

$$\begin{aligned} \Lambda_V(\hat{\theta}_v, \sigma_p) &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V + \int_{\hat{\theta}_v}^{\bar{\theta}} \hat{\theta}_v dG_V \\ &= \hat{\theta}_v - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v, \end{aligned} \tag{63}$$

and the monitoring cost is

$$\begin{aligned} \gamma\Omega_V(\hat{\theta}_v \sigma_p) &= \gamma \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v dG_V \\ &= \gamma \left[\hat{\theta}_v G_V(\hat{\theta}_v, \sigma_p) - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v\right]. \end{aligned} \tag{64}$$

Then (62) can be written as

$$Z(.) = \hat{\theta}_v \gamma \left[G_V(\hat{\theta}_v, \sigma_p^A) - G_V(\hat{\theta}_v, \sigma_p^B)\right] + (1-\gamma) \int_{\underline{\theta}}^{\hat{\theta}_v} \left[G_V(\theta_v, \sigma_p^A) - G_V(\theta_v, \sigma_p^B)\right] d\theta_v. \tag{65}$$

Since $G_V(\theta_v, \sigma_p^A)$ and $G_V(\theta_v, \sigma_p^B)$ are MPS of $G(\hat{\theta})$ with a higher shifting parameter in country B, it follows:

(i) Capital producers in country B have a higher default probability at a given cutoff point, i.e. $G_V(\hat{\theta}_v, \sigma_p^A) < G_V(\hat{\theta}_v, \sigma_p^B)$. Thus, the first term of (65) is negative.

(ii) From (43), $G_V(\theta_v, \sigma_p^A)$ SOSD $G_V(\theta_v, \sigma_p^B)$, so the second term is also negative.

In sum, the first part of the Proposition is proved as $Z(\hat{\theta}_v, \sigma_p^A, \sigma_p^B) < 0$. With this result and the definition of the gross return on investment, the second part follows immediately:

$$GRI_V(\hat{\theta}_v, \sigma_p^B) = \frac{\alpha}{\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^B)} > \frac{\alpha}{\Lambda_V(\hat{\theta}_v, \sigma_p^A) - \gamma\Omega_V(\hat{\theta}_v, \sigma_p^A)} = GRI_V(\hat{\theta}_v, \sigma_p^A). \quad (66)$$

□

As a result, when the aggregate volatility decreases, the GRI function shifts down at any level of $\hat{\theta}_v$. Before checking the risk premium function, let's establish the following properties that will be used next.

Proposition 4.2. *For a given $\hat{\theta}_v$, the lower is the aggregate risk (σ_p), the higher is the expected gross payment $\Lambda_V(\hat{\theta}_v, \sigma_p)$ and the lower is the expected monitoring cost $\gamma\Omega_V(\hat{\theta}_v, \sigma_p)$, both relative to the expected income in each state. Furthermore, the $\lambda_v(\hat{\theta}_v, \sigma_p)$ decreases when the aggregate uncertainty is lower.*

Proof. See Appendix D. □

With the previous results, it is possible to find the effect of reducing the output fluctuation on the risk premium function:

Proposition 4.3. *For a given cut off point, the lower is the aggregate risk (σ_p), the lower is the risk premium: $\varepsilon_V(\hat{\theta}_v, \sigma_p^A) < \varepsilon_V(\hat{\theta}_v, \sigma_p^B)$ if $\sigma_p^B > \sigma_p^A$.*

Proof. (The arguments of the functions are omitted for simplicity) If the Prosposition is true, then $\left[\varepsilon_V^A - \varepsilon_V^B\right] < 0$. From (56)

$$\varepsilon_V^A - \varepsilon_V^B = \frac{\lambda_v^A}{\left(1 - \Lambda_V^A\right) + \lambda_v^A\left(\Lambda_V^A - \gamma\Omega_V^A\right)} - \frac{\lambda_v^B}{\left(1 - \Lambda_V^B\right) + \lambda_v^B\left(\Lambda_V^B - \gamma\Omega_V^B\right)}$$

$$= \frac{\lambda_v^A\left[\left(1 - \Lambda_V^B\right) + \lambda_v^B\left(\Lambda_V^B - \gamma\Omega_V^B\right)\right] - \lambda_v^B\left[\left(1 - \Lambda_V^A\right) + \lambda_v^A\left(\Lambda_V^A - \gamma\Omega_V^A\right)\right]}{\left[\left(1 - \Lambda_V^A\right) + \lambda_v^A\left(\Lambda_V^A - \gamma\Omega_V^A\right)\right]\left[\left(1 - \Lambda_V^B\right) + \lambda_v^B\left(\Lambda_V^B - \gamma\Omega_V^B\right)\right]}. \quad (67)$$

Since the denominator of (67) is always positive, the sign of the ratio is determined by the numerator,

$$sign\left(\varepsilon_V^A - \varepsilon_V^B\right) = sign\left\{\lambda_v^A\left[\left(1-\Lambda_V^B\right)+\lambda_v^B\left(\Lambda_V^B-\gamma\Omega_V^B\right)\right] - \lambda_v^B\left[\left(1-\Lambda_V^A\right)+\lambda_v^A\left(\Lambda_V^A-\gamma\Omega_V^A\right)\right]\right\}.$$

After adding and subtracting $\lambda_v^B\Lambda_V^B$ to the numerator, collect terms to show

$$sign\left(\varepsilon_V^A - \varepsilon_V^B\right) = sign\left\{\left(\lambda_v^A-\lambda_v^B\right)\left(1-\Lambda_V^B\right)+\left(\lambda_v^A-1\right)\lambda_v^B\left(\Lambda_V^B-\Lambda_V^A\right) + \gamma\lambda_v^A\lambda_v^B\left(\Omega_V^A-\Omega_V^B\right)\right\}.$$

From Proposition 4.2, we know that, for a given $\hat{\theta}_v$, $\left(\lambda_v^A-\lambda_v^B\right)<0$, $\left(\Lambda_V^B-\Lambda_V^A\right)<0$, and $\left(\Omega_V^A-\Omega_V^B\right)<0$. Given that $\lambda_v>1$ and $(1-\Lambda_V)>0$ for any σ_p, the $sign\left(\varepsilon_V^A-\varepsilon_V^B\right)$ is always negative when $\sigma_p^B>\sigma_p^A$. In words, the reduction in aggregate volatility lowers the risk premium. □

Therefore, the function $\varepsilon_V(\hat{\theta}_v,\sigma_p^B)$ will also shift down as the aggregate volatility decreases.

Although a reduction in the volatility lowers both sides of (61), reaching a new equilibrium at a lower risk premium, the effect on the optimal break-even point is ambiguous.[23] As is shown in Figure 3, the final effect on $\hat{\theta}_v^*$ depends on the size of the shift of each curve. That is, if the expected return on investment drops more than the risk premium, the producer chooses a relatively lower cutoff point that decreases the risk premium (Figure 3a). Conversely, when the risk premium decreases more than the return on investment, the producer can choose a higher cutoff point to close the gap (Figure 3b). The last case might be a possible result, since the value at which the producer's expected payment reaches a global maximum is higher when the aggregate risk is lower.[24]

Even though one would expect that $\Delta\varepsilon_V(\hat{\theta}_v,\sigma_p)>\Delta GRI_V(\hat{\theta}_v,\sigma_p)$, nothing guarantees

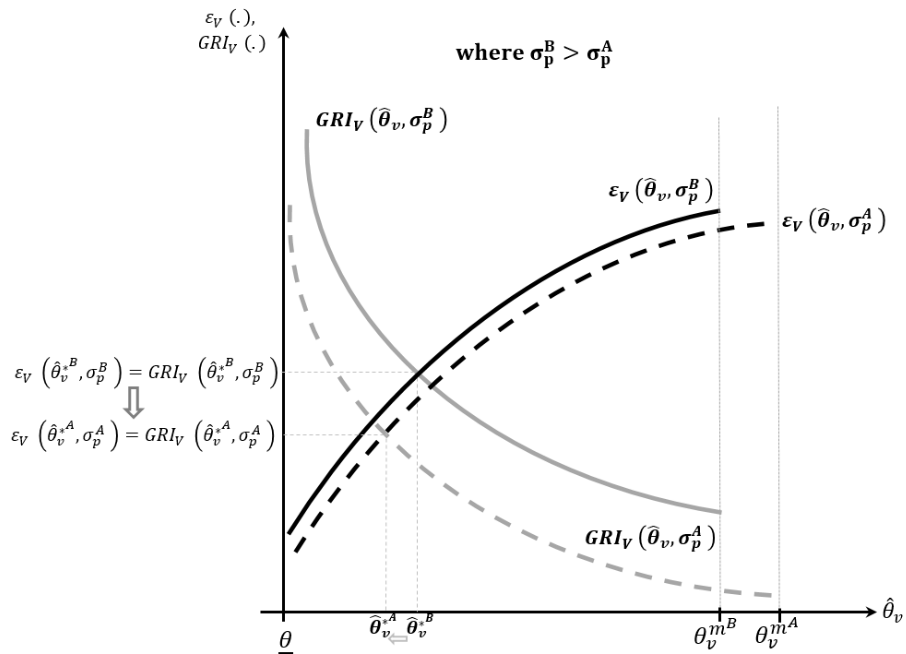
[23] It is possible to show that the conditions to apply the implicit function theorem in $F(\hat{\theta}_v^*,\sigma_p)=\varepsilon_V(\hat{\theta}_v^*,\sigma_p)-GRI_V(\hat{\theta}_v^*,\sigma_p)=0$ are satisfied, including $F'_{\hat{\theta}_v^*}\neq 0$.

[24] The global maximum of $[\Lambda_V(\hat{\theta}_v,\sigma_p^B)-\gamma\Omega_V(\hat{\theta}_v,\sigma_p^B)]$ is reached when $\Lambda'_V(\theta_v^{m^B},\sigma_p^B)=\gamma\Omega'_V(\theta_v^{m^B},\sigma_p^B)$. Given that $\partial\Lambda'_V/\partial\hat{\theta}_v<0$ and $\partial\Omega'_V/\partial\hat{\theta}_v>0$, and that Λ'_V increases and Ω'_V decreases when the aggregate risk declines (see Appendix D, Claim 4), then, the new maximizer must satisfy $\theta_v^{m^A}>\theta_v^{m^B}$.

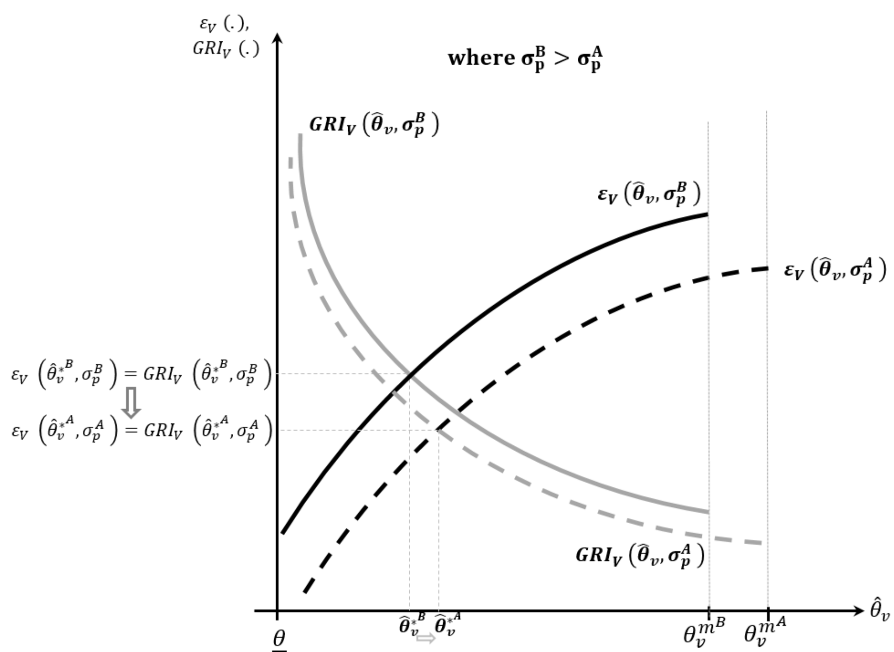
this outcome. If both functions change in the same proportion, the optimal cutoff point will be unaffected. In any case, the resulting optimal value of $\hat{\theta}_v^{*A}$ will not affect the main result, because the risk premium is always lower in an economy with a lower σ_p.

Figure 3: Aggregate Risk Reduction: Relative Changes in ε_V and GRI_V

(a) If $\Delta\varepsilon_V(\hat{\theta}_v, \sigma_p) < \Delta GRI_V(\hat{\theta}_v, \sigma_p)$



(b) If $\Delta\varepsilon_V(\hat{\theta}_v, \sigma_p) > \Delta GRI_V(\hat{\theta}_v, \sigma_p)$



Finally, if the steady state level of investment before and after the reduction in aggregate volatility are compared for the same R^f and k_0, we get

$$l_v^{*B} = \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^{*B}, \sigma_p^B)R^f} \right]^{\frac{1}{1-\alpha\delta}} < \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^{*A}, \sigma_p^A)R^f} \right]^{\frac{1}{1-\alpha\delta}} = l_v^{*A}, \tag{68}$$

as $\varepsilon_V(\hat{\theta}_v^{*A}, \sigma_p^A) < \varepsilon_V(\hat{\theta}_v^{*B}, \sigma_p^B)$, regardless of $\hat{\theta}_v^{*A} \lesseqgtr \hat{\theta}_v^{*B}$.

The previous finding is summarized in the following statement for an interior solution:

Result 3. *Everything else equal, a decrease (increase) in aggregate risk (σ_p) will reduce (increase) the risk premium (ε_V), increasing (decreasing) the equilibrium level of loans to the private sector (l_v^*), which is equivalent to saying that investment will be higher (lower).*

Since $B_v^{d*} = d_v^* - l_v^*$, the following Corollary holds:

Corollary 1. *Given the assumptions in this section, the demand for government bonds in steady state (B_v^{d*}) goes up as the aggregate volatility increases and the current state is high ($S_t = H$). However, if $S_t = L$, the change in B_v^{d*} is ambiguous, and it depends on the relative drop of deposits vis-à-vis the change of loans to the private sector.*

In this model, the government bond is a risk-free asset that does not require monitoring, thus it is a natural refuge in a riskier environment as the aggregate volatility increases. However, government bonds in emerging economies are subject to aggregate risk. If the return of government bonds were affected by σ_p or σ_y, the financial intermediary would have to allocate funds between positively correlated risky assets without any safety choice. However, in many developing countries, the domestic financial institutions are willing to absorb risky government bonds by reducing the credits to private sector. If this is the case, introducing risk in the goverment debt would only generate a level effect.[25]

Combining Results 1 and 3, and substitution (58) in equation (59), it is possible to show that output volatility (σ_y) is a nonmonotonic function with respect to the aggregate risk parameter. For instance, a reduction in σ_p tends to decrease the output volatility, but it simultaneously boosts private investment, which tends to increase the output fluctuations.

[25] The financial system in these economies is populated by the so-called "lazy banks". Emran and Farazi (2009) find evidence of a "lazy" bank behavior in some developing countries.

Formally,

$$\frac{\partial \sigma_y}{\partial \sigma_p} = \varPi \left[\frac{\alpha\delta E\left[A(S)\right]}{\varepsilon_V(\hat{\theta}_v^*, \sigma_p) R^f} \right]^{\frac{\alpha\delta}{1-\alpha\delta}} \lesseqgtr 0, \tag{69}$$

where $\varPi = 1 - \sigma_p \left(\frac{\alpha\delta}{1-\alpha\delta}\right) \left[\frac{\partial \varepsilon_V(\cdot)/\partial \sigma_p}{\varepsilon_V(\cdot)}\right]$. Since the second factor of (69) is always positive, the effect of a change in aggregate risk will depend on the $sign(\varPi)$. Let's define the elasticity of the risk premium with respect to the aggregate risk:

$$\xi_{\varepsilon_V, \sigma_p} = \frac{\sigma_p}{\varepsilon_V(\cdot)} \left[\frac{\partial \varepsilon_V(\cdot)}{\partial \sigma_p} \right]. \tag{70}$$

Therefore, $\varPi$ can be written as

$$\varPi = 1 - \xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta} \right]. \tag{71}$$

The sign of (71) depends of the elasticity of the risk premium and the value of $(\alpha\delta)$. The latter is the investment rate (l/y) weighted by the marginal product of investment (MPl), which is usually lower or equal than 1/2 for reasonable values of the capital share in total income (δ) and the curvature of the production function of capital goods (α). Then the expression $\left[\frac{\alpha\delta}{1-\alpha\delta}\right]$ is usually less or equal to one. From Result 3, $\partial \varepsilon_V(\cdot)/\partial \sigma_p > 0$, thus, $\xi_{\varepsilon_V, \sigma_p}$ is always positive. Given these results, three cases can be analyzed:

(i) If $\xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta}\right] < 1$, then (69) is always positive, $\frac{\partial \sigma_y}{\partial \sigma_p} > 0$. That is, an increase (decrease) in the aggregate risk, increases (reduces) the output volatility, but at a lower amount due to the reduction (increase) in the aggregate investment level.

(ii) If $\xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta}\right] = 1$, then $\frac{\partial \sigma_y}{\partial \sigma_p} = 0$, which means that the effect of the aggregate risk on output is perfectly compensated by the change in investment.

(iii) Finally, if $\xi_{\varepsilon_V, \sigma_p} \left[\frac{\alpha\delta}{1-\alpha\delta}\right] > 1$, then (69) is always negative, $\frac{\partial \sigma_y}{\partial \sigma_p} < 0$: the effect of the aggregate risk on output volatility is more than compensated by the change in investment.

In general $\frac{\alpha\delta}{1-\alpha\delta} \leq 1$, so that cases (ii) and (iii) are only possible for high values of $\xi_{\varepsilon_V, \sigma_p}$.

In other words, the more sensitive is the risk premium to changes in the aggregate risk, the lower is the direct effect of the latter on the output volatility. For relatively high values of elasticity, significant adjustments of the risk premium would lead to changes in the investment levels that more than compensate for the effects of aggregate uncertainty on output volatility, resulting in counterintuitive results. Appendix E shows different combinations of elasticities ($\xi_{\varepsilon_V,\sigma_p}$), α, and δ to check for the range of values for these variables where cases (ii) and (iii) are possible.

Since the possible effects of aggregate risk on optimal investment and output volatility are defined, Result 1 can be replaced by

Result 4. *In this framework, changes in the aggregate risk (σ_p) do not translate one-to-one into the output volatility (σ_y), since they are partially compensated by their effect on the investment level (l_v^*). For significantly high values of the risk-premium elasticity with respect to the aggregate risk, the change in investment more than compensates the direct impact of the change in aggregate uncertainty on output volatility, resulting in a null effect or in an opposite-direction variation of σ_y.*

4.2 Financial Depth after a Change in Aggregate Volatility

When the aggregate risk or TFP volatility decreases, keeping constant the $E[A(S)]$, the financial depth indicator (60) is affected by two forces. From Result 3, as the risk premium decreases, the amount of loans to the private sector increases, which tends to raise the FD indicator. On the other hand, as is stated in Result 2, the relative change in the current productivity shock generates an ambiguous effect on FD. Continuing with the previous exercise, the financial depth indicator in steady state (for a given R^f) in the economies B and A are

$$FD_v^{*^B} = \left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^{*^B}, \sigma_p^B)R^f}\right]\left[\frac{E\left[A(S)\right]}{A(S^b)}\right] \tag{72}$$

and

$$FD_v^{*A} = \left[\frac{\alpha\delta}{\varepsilon_V(\hat{\theta}_v^{*A}, \sigma_p^A) R^f} \right] \left[\frac{E\left[A(S)\right]}{A(S^a)} \right]. \tag{73}$$

By combining (72) and (73), a reduction in the aggregate risk will increase the financial depth if and only if:

$$\frac{\varepsilon_V(\hat{\theta}_v^{*B}, \sigma_p^B)}{\varepsilon_V(\hat{\theta}_v^{*A}, \sigma_p^A)} > \frac{A(S^a)}{A(S^b)}. \tag{74}$$

However, when volatility goes down keeping constant the mean, it is like a mean-preserving shift of the two possible productivity shocks. If $\sigma_p^B > \sigma_p^A$ with $E[A(S^b)] = E[A(S^a)]$, it must be the case that with equal probabilities

$$A(H^b) > A(H^a) > A(L^a) > A(L^b) > 0. \tag{75}$$

If $\sigma_p^B > \sigma_p^A$, then $\frac{\varepsilon_V(\hat{\theta}_v^{*B}, \sigma_p^B)}{\varepsilon_V(\hat{\theta}_v^{*A}, \sigma_p^A)} > 1$ (Result 3). Therefore, condition (74) is automatically satisfied in the high state ($S = H$), because $\frac{A(H^a)}{A(H^b)} < 1$. But when the aggregate state is low ($S = L$), condition (74) does not automatically hold, since $\frac{A(L^a)}{A(L^b)} > 1$. In sum, one can summarize the previous findings as follows

Result 5. *Everything else equal, and under the assumption of this model, a decrease in aggregate risk (σ_p) will "automatically" increase the financial depth indicator (FD) in the high state ($S = H$). However, in the low state ($S = L$), FD increases if condition (74) holds. That is, when the relative reduction in the risk premium (ε_V) more than compensate the relative increase in aggregate productivity during bad times.*

Notice that if the financial depth indicator is calculated with respect to the expected output, from simplifying (72) and (73), it is straightforward to show that

Result 6. *Everything else equal, a decrease in the aggregate risk (σ_p) will always increase the financial depth indicator (FD) when is calculated with respect to the expected output, given the reduction in the risk premium (Result 3).*

5 Conclusions and Possible Extensions

This chapter has been devoted to analyze different effects of aggregate risk (TFP shocks) on output volatility, measured by the standard deviation of output, and the financial depth, as poxied by the credits to private sector relative to output. Several empirical works study how the increase in financial development can reduce the output volatility by using different estimation techniques. However, I have followed the approach of a different strand of theoretical and empirical literature, where high volatility is the result of a concentrated production patterns, and is ex-ante independent of the financial system.

Through a simplified theoretical model, I find that a mean preserving change in the aggregate productivity shocks can affect financial depth. That is, a reduction (increase) in the aggregate risk not only reduces (raises) the output volatility for reasonable values of the risk-premium elasticity with respect to aggregate uncertainty (see Result 4), but it may increase (reduce) the financial depth indicator (Results 3, 5, and 6). These findings do not rule out the possibility that, ex-post, a deeper financial system can help in reducing the macro fluctuations, but they show that a shallow financial system is a by-product of the aggregate volatility. Therefore, it must be the case that for some emerging countries with high and persistent aggregate volatility, the development of a deeper financial system is restricted by the real volatility. Since the change in production patterns and technologies require time and resources, policy makers should implement more countercyclical macroeconomic policies in the meantime to lessen the aggregate fluctuations and trigger the beneficial effects on the financial system.

From a theoretical perspective, the model is flexible enough to introduce additional features of developing economies. For example, one might explore different policy interventions, such as an exogenous policy for government spending. Following with the public sector, it would be interesting to relax the assumption of a perfectly elastic supply of bonds, endogenizing the risk-free rate. Another possible extension is to open the economy by allowing domestic agents to borrow from the global financial system. If there is an external risk-free asset, the domestic government bonds might be affected by the aggregate volatility. Given the difficulties to get credit in some emerging countries, capital-producers could also use a mix of self-funding and loans to finance investment opportunities.

From an empirical point of view, one should find evidence that supports Corollary 1, analyzing the exposure of the domestic financial system to government debt as the macroeconomic volatility increases/decreases. The relationship between aggregate volatlity and financial depth can be estimated through a panel VAR to account for possible interactions. Finally, the level of concentration of domestic production could be introduced in the analysis by using a Herfindahl-Hirschman index to test theoretical predictions.

Further analysis is required to answer the question of how high and persistent macroeconomic fluctuations may hinder the financial deepening in economies affected by intrinsic volatility volatility originating in the real sectors. This study contributes by uncovering some economic channels to address this important question.

References

Acemoglu, D., S. Johnson, J. Robinson, and Y. Thaicharoen (2003). Institutional causes, macroeconomic symptoms: volatility, crises and growth. *Journal of Monetary Economics 50*(1), 49–123.

Acemoglu, D. and F. Zilibotti (1997). Was Prometheus unbound by chance? risk, diversification, and growth. *Journal of Political Economy 105*(4), 709–751.

Aghion, P., G.-M. Angeletos, A. Banerjee, and K. Manova (2010). Volatility and growth: Credit constraints and the composition of investment. *Journal of Monetary Economics 57*(3), 246–265.

Aghion, P., A. Banerjee, and T. Piketty (1999). Dualism and macroeconomic volatility. *The Quarterly Journal of Economics 114*(4), 1359–1397.

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy 115*(1), 69–102.

Bansal, R., V. Khatchatrian, and A. Yaron (2005). Interpretable asset markets? *European Economic Review 49*(3), 531–560.

Beck, T., M. Lundberg, and G. Majnoni (2006). Financial intermediary development and growth volatility: Do intermediaries dampen or magnify shocks? *Journal of International Money and Finance 25*(7), 1146–1167.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *American Economic Review 79*(1), 14–31.

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics 1*, 1341–1393.

Braun, M. and B. Larrain (2005). Finance and the business cycle: international, inter-industry evidence. *The Journal of Finance 60*(3), 1097–1128.

Calderón, C., P. Chuhan-Pole, and R. M. López-Monti (2017). Cyclicality of fiscal policy in Sub-Saharan Africa. *Policy Research Working Paper; No. 8108*. World Bank, Washington, DC.

Čihák, M., A. Demirgüç-Kunt, E. Feyen, and R. Levine (2012). Benchmarking financial systems around the world. *World Bank Policy Research Working Paper 6175*. World Bank, Washington, DC.

Diamond, P. A. and J. E. Stiglitz (1974). Increases in risk and in risk aversion. *Journal of Economic Theory 8*(3), 337–360.

Emran, M. S. and S. Farazi (2009). Lazy banks? government borrowing and private credit in developing countries. *Institute for International Economic Policy Working Paper 9*.

Gale, D. and M. Hellwig (1985). Incentive-compatible debt contracts: The one-period problem. *The Review of Economic Studies 52*(4), 647–663.

Greenwood, J. and B. Jovanovic (1990). Financial development, growth, and the distribution of income. *Journal of Political Economy 98*(5, Part 1), 1076–1107.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*(2), 211–248.

Koren, M. and S. Tenreyro (2007). Volatility and development. *The Quarterly Journal of Economics 122*(1), 243–287.

Koren, M. and S. Tenreyro (2013). Technological diversification. *American Economic Review 103*(1), 378–414.

Kraay, A. and J. Ventura (2007). Comparative advantage and the cross-section of business cycles. *Journal of the European Economic Association 5*(6), 1300–1333.

Labadie, P. (1998). Aggregate fluctuations, financial constraints and risk sharing. *Economic Theory 12*(3), 621–648.

Lettau, M., S. C. Ludvigson, and J. A. Wachter (2007). The declining equity premium: What role does macroeconomic risk play? *The Review of Financial Studies 21*(4), 1653–1687.

Raddatz, C. (2006). Liquidity needs and vulnerability to financial underdevelopment. *Journal of Financial Economics 80*(3), 677–722.

Rothschild, M. and J. E. Stiglitz (1970). Increasing risk: I. a definition. *Journal of Economic Theory 2*(3), 225–243.

Rothschild, M. and J. E. Stiglitz (1972). Addendum to "Increasing risk: I. a definition". *Journal of Economic Theory 5*(2), 306–306.

Sahay, M. R., M. Cihak, M. P. M. N'Diaye, M. A. Barajas, M. D. B. A. Pena, R. Bi, M. Gao, M. A. J. Kyobe, and L. Nguyen (2015). Rethinking financial deepening: Stability and growth in emerging markets. *IMF Staff Discussion Note*. International Monetary Fund.

Uhlig, H. (1996). A law of large numbers for large economies. *Economic Theory 8*(1), 41–50.

Wang, P., Y. Wen, and Z. Xu (2018). Financial development and long-run volatility trends. *Review of Economic Dynamics 28*, 221–251.

APPENDIX

A Benchmark Model: Some Comparative Statics

Figure A1: An Increase in the Risk-Free Rate of Return

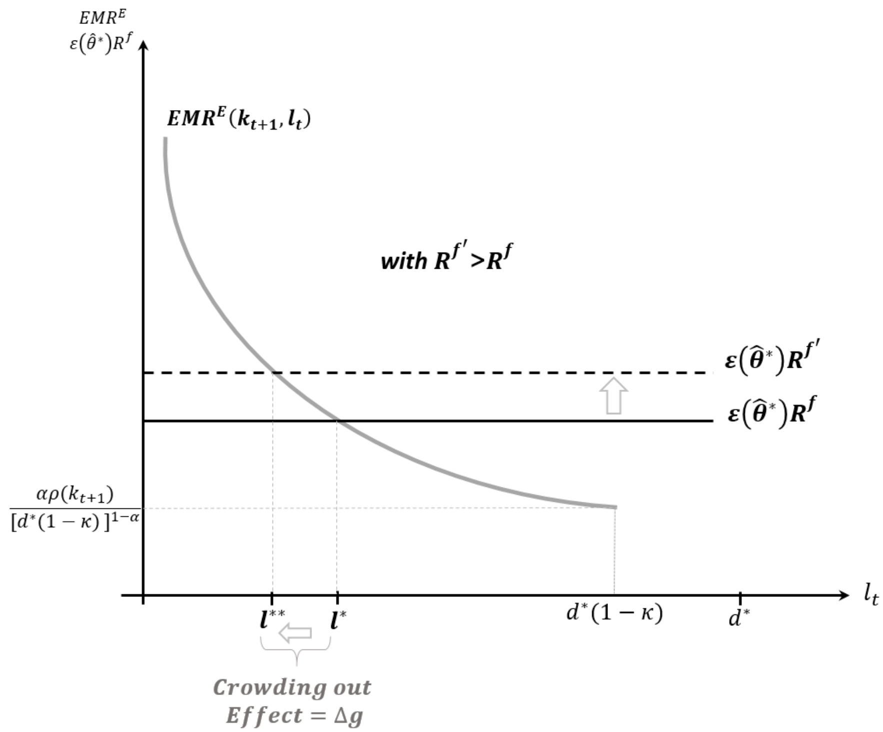


Figure A2: An Increase in the Excess Return

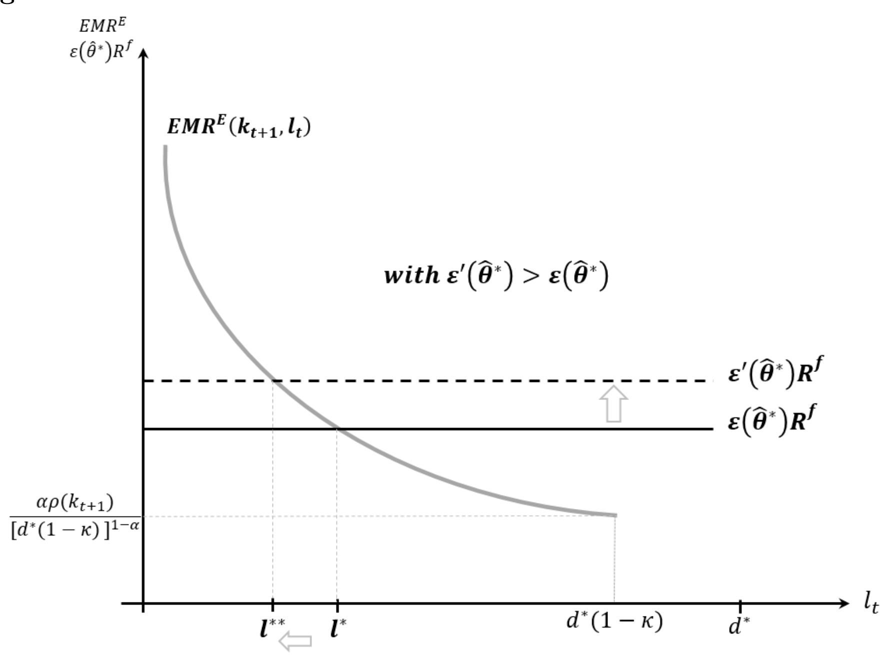
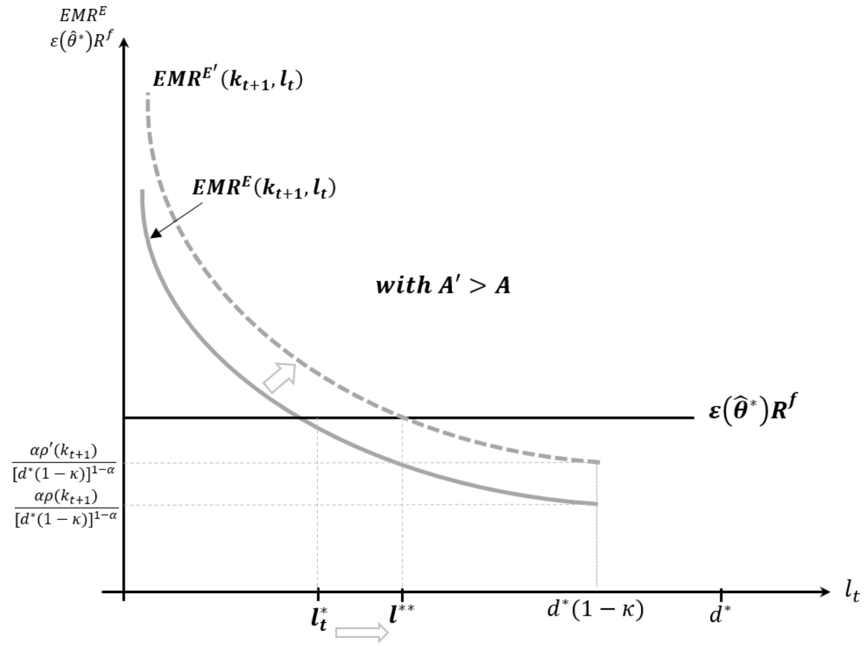


Figure A3: An Increase in the Total Factor Productivity



B Benchmark Model: Second-Order Condition

To satisfy the sufficient condition for a local maximum, the corresponding Bordered Hessian matrix (B) must be the negative definite. Since the sign of the first leading principal minor is always negative, it is sufficient to check the sign of the determinant of the entire Bordered Hessian defined as follows:

$$\det B = \begin{vmatrix} 0 & \mathcal{L}_{\lambda,\hat{\theta}} & \mathcal{L}_{\lambda,l_t} \\ \mathcal{L}_{\lambda,\hat{\theta}} & \mathcal{L}_{\hat{\theta},\hat{\theta}} & \mathcal{L}_{\hat{\theta},l_t} \\ \mathcal{L}_{\lambda,l_t} & \mathcal{L}_{\hat{\theta},l_t} & \mathcal{L}_{l_t,l_t} \end{vmatrix}_{(\boldsymbol{\hat{\theta}^*}, \boldsymbol{l^*})}$$

All partial derivatives in this matrix are evaluated at the critical values $(\hat{\theta}^*, l^*)$.

Claim 1. *If* $\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta}) < 0$ *then the Bordered Hessian matrix (B) associated with the maximization of (20) is negative definite. Therefore, a (local) maximum is achieved at* $(\boldsymbol{\hat{\theta}^*}, \boldsymbol{l^*})$.

Proof. Recall the determinant is

$$\det B = \mathcal{L}_{\lambda,\hat{\theta}}\mathcal{L}_{l_t,\hat{\theta}}\mathcal{L}_{\lambda,l_t} + \mathcal{L}_{\lambda,l_t}\mathcal{L}_{\lambda,\hat{\theta}}\mathcal{L}_{\hat{\theta},l_t} - \left[(\mathcal{L}_{\lambda,l_t})^2\mathcal{L}_{\hat{\theta},\hat{\theta}} + \mathcal{L}_{l_t,l_t}(\mathcal{L}_{\lambda,\hat{\theta}})^2\right], \tag{76}$$

where:

$$\mathcal{L}_{\lambda,\hat{\theta}} = \rho(k_{t+1})l_t^{\alpha}\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right], \tag{77}$$

$$\mathcal{L}_{\lambda,l_t} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right] - R^f_{t+1}, \tag{78}$$

$$\mathcal{L}_{\hat{\theta},\hat{\theta}} = \rho(k_{t+1})l_t^{\alpha}\left\{\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta})\right\}, \tag{79}$$

$$\mathcal{L}_{\hat{\theta},l_t} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left\{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\lambda - \Lambda'(\hat{\theta})\right\}, \tag{80}$$

$$\mathcal{L}_{l_t,\hat{\theta}} = \alpha\rho(k_{t+1})l_t^{\alpha-1}\left\{-\Lambda'(\hat{\theta}) + \lambda\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\right\}, \tag{81}$$

$$\mathcal{L}_{l_t,l_t} = \alpha(\alpha-1)\rho(k_{t+1})l_t^{\alpha-2}\left\{\left[1 - \Lambda(\hat{\theta})\right] + \lambda\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]\right\}. \tag{82}$$

From the FOC (21a) $\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\lambda - \Lambda'(\hat{\theta}) = 0$, thus, $\mathcal{L}_{\hat{\theta},l_t} = \mathcal{L}_{l_t,\hat{\theta}} = 0$ and (76) can be written as

$$\det B = -(\mathcal{L}_{\lambda,l_t})^2\mathcal{L}_{\hat{\theta},\hat{\theta}} - \mathcal{L}_{l_t,l_t}(\mathcal{L}_{\lambda,\hat{\theta}})^2. \tag{83}$$

To be negative definite the det B must be positive. Since $\mathcal{L}_{l_t,l_t} < 0$, the sign of (83) depends on the sign of $\mathcal{L}_{\hat{\theta},\hat{\theta}}$, which is negative iff $\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\lambda - \Lambda''(\hat{\theta}) < 0$. □

C Properties of the Excess Return at the Cutoff Point

Since $0 < \Lambda(\hat{\theta}) < 1$ and $\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) > 0$ given that $\hat{\theta}^* \in (\underline{\theta}, \theta^m)$, it must be that $0 < \Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta}) < 1$. As $\Lambda'(\hat{\theta}) > 0$ and $(1 - \Lambda'(\hat{\theta})) > 0$, then $\varepsilon(\hat{\theta}) > 0$ and by contradiction is straightforward to prove that [26]

$$\varepsilon(\hat{\theta}) = \frac{\Lambda'(\hat{\theta})}{\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]} > 1. \tag{84}$$

Taking derivative with respect to the break-even point:

$$\varepsilon'(\hat{\theta}) = \frac{\Lambda''(\hat{\theta})D(\hat{\theta}) - D'(\hat{\theta})\Lambda'(\hat{\theta})}{\left[D(\hat{\theta})\right]^2}, \tag{85}$$

where

$$D(\hat{\theta}) = \left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda'(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right], \tag{86}$$

$$D'(\hat{\theta}) = \left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\left[1 - \Lambda(\hat{\theta})\right] + \Lambda''(\hat{\theta})\left[\Lambda(\hat{\theta}) - \gamma\Omega(\hat{\theta})\right]. \tag{87}$$

The sign of $\varepsilon'(\hat{\theta})$ is defined by the sign of the numerator of (85). Substituting (86) and (87) in the numerator and simplifying terms I get

$$\begin{aligned}\Lambda''(\hat{\theta})D(\hat{\theta}) - D'(\hat{\theta}) = &\left[1 - \Lambda(\hat{\theta})\right]\left\{\Lambda''(\hat{\theta})\left[\Lambda'(\hat{\theta}) - \gamma\Omega'(\hat{\theta})\right] - \Lambda'(\hat{\theta})\left[\Lambda''(\hat{\theta}) - \gamma\Omega''(\hat{\theta})\right]\right\}\\ =&\gamma\left[1 - \Lambda(\hat{\theta})\right]\left[\Lambda'(\hat{\theta})\Omega''(\hat{\theta}) - \Lambda''(\hat{\theta})\Omega'(\hat{\theta})\right].\end{aligned}$$

Since $\Lambda'(\hat{\theta}) > 0$ and $\Lambda''(\hat{\theta}) < 0$, while both $\Omega'(\hat{\theta})$ and $\Omega''(\hat{\theta})$ are positive, the numerator of (85) is positive. Therefore, $\varepsilon(\hat{\theta})$ is increasing in the cutoff point.

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[26] If $0 < \varepsilon(\hat{\theta}) < 1$ then $\left[-\gamma\Omega'(\hat{\theta})\left[1 - \Lambda(\hat{\theta})\right] - \gamma\Lambda'(\hat{\theta})\Omega(\hat{\theta})\right] > 0$, which is not possible given the properties of $\Lambda(\cdot)$ and $\Omega(\cdot)$. Thus, the risk premium must be greater than 1.

D Proof of Preposition 4.2

To prove the Preposition, let's consider a change in the aggregate risk such as $\sigma_p^B > \sigma_p^A$. Then, the following properties hold:

Claim 2. *For a given $\hat{\theta}_v$, the expected gross payment, relative to the expected income, is higher in economy A, then $\left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \Lambda_V(\hat{\theta}_v, \sigma_p^A)\right] < 0$.*

Proof. In general, $\Lambda_V(\hat{\theta}_v, \sigma_p) = \hat{\theta}_v - \int_{\underline{\theta}}^{\hat{\theta}_v} G_V(\theta_v, \sigma_p) d\theta_v$ for any σ_p, then

$$\left[\Lambda_V(\hat{\theta}_v, \sigma_p^B) - \Lambda_V(\hat{\theta}_v, \sigma_p^A)\right] = \int_{\underline{\theta}}^{\hat{\theta}_v} \left[G_V(\theta_v, \sigma_p^A) - G_V(\theta_v, \sigma_p^B)\right] d\theta_v, \tag{88}$$

which is negative since $G_V(\theta_v, \sigma_p^A)$ SOSD $G_V(\theta_v, \sigma_p^B)$. □

Claim 3. *For a given $\hat{\theta}_v$, the expected monitoring cost, relative to the expected income, is higher in economy B, then $\gamma \left[\Omega_V(\hat{\theta}_v, \sigma_p^A) - \Omega_V(\hat{\theta}_v, \sigma_p^B)\right] < 0$.*

Proof. This result comes directly from (Assumption 4):
From equation (53), $\Omega_V(\hat{\theta}_v, \sigma_p) = \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p) d\theta_v$, thus

$$\begin{aligned}\left[\Omega_V(\hat{\theta}_v, \sigma_p^A) - \Omega_V(\hat{\theta}_v, \sigma_p^B)\right] &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p^A) d\theta_v - \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v g_V(\theta_v, \sigma_p^B) d\theta_v \\ &= \int_{\underline{\theta}}^{\hat{\theta}_v} \theta_v \left[g_V(\theta_v, \sigma_p^A) - g_V(\theta_v, \sigma_p^B)\right] d\theta_v < 0, \end{aligned} \tag{89}$$

since the mean-preserving increase in risk will shift up the density function over the interval $[\underline{\theta}, \hat{\theta}_v]$ □

Claim 4. *For a given* $\hat{\theta}_v$, $\left[\lambda_v(\hat{\theta}_v, \sigma_p^A) - \lambda_v(\hat{\theta}_v, \sigma_p^B)\right] < 0$.

Proof. To simplify notation, the arguments of the functions are omitted. From the FOC (55a)

$$\begin{aligned}\left[\lambda_v^A - \lambda_v^B\right] &= \frac{\Lambda_V^{\prime A}}{\Lambda_V^{\prime A} - \gamma\Omega_V^{\prime A}} - \frac{\Lambda_V^{\prime B}}{\Lambda_V^{\prime B} - \gamma\Omega_V^{\prime B}} \\ &= \frac{\Lambda_V^{\prime A}\left[\Lambda_V^{\prime B} - \gamma\Omega_V^{\prime B}\right] - \Lambda_V^{\prime B}\left[\Lambda_V^{\prime A} - \gamma\Omega_V^{\prime A}\right]}{\left[\Lambda_V^{\prime A} - \gamma\Omega_V^{\prime A}\right]\left[\Lambda_V^{\prime B} - \gamma\Omega_V^{\prime B}\right]}. \end{aligned} \tag{90}$$

The denominator of (90) is positive, so the sign of the ratio is defined by the numerator, which can be written as $\gamma\left[\Lambda^{\prime B}\Omega_V^{\prime A} - \Lambda^{\prime A}\Omega_V^{\prime B}\right]$. By adding and subtracting $\gamma\Lambda^{\prime B}\Omega_V^{\prime B}$ to the numerator, the sign is determined by

$$sign\left(\lambda_v^A - \lambda_v^B\right) = sign\left\{\Lambda_V^{\prime B}\left[\Omega_V^{\prime A} - \Omega_V^{\prime B}\right] + \Omega_V^{\prime B}\left[\Lambda_V^{\prime B} - \Lambda_V^{\prime A}\right]\right\}. \tag{91}$$

Given that $\Lambda_V^{\prime} > 0$ and $\Omega_V^{\prime} > 0$, we just need to find the signs of the changes in marginal (expected) gross payment and monitoring cost. Since $\Lambda_V^{\prime} = 1 - G_V(\hat{\theta}_v, \sigma_p)$, then

$$\left[\Lambda_V^{\prime B} - \Lambda_V^{\prime A}\right] = G_V(\hat{\theta}_v, \sigma_p^A) - G_V(\hat{\theta}_v, \sigma_p^B) < 0 \quad \text{(from Assumption 4)}. \tag{92}$$

While $\Omega_V^{\prime} = \hat{\theta} g_V(\hat{\theta}_v, \sigma_p)$

$$\left[\Omega_V^{\prime A} - \Omega_V^{\prime B}\right] = \hat{\theta}\left[g_V(\hat{\theta}_v, \sigma_p^A) - g_V(\hat{\theta}_v, \sigma_p^B)\right] < 0 \quad \text{(from Assumption 4)}. \tag{93}$$

Consequently, $\left(\lambda_v^A - \lambda_v^B\right) < 0$. □

E Sensitivity Analysis for Different Values of Elasticities ($\xi_{\varepsilon_V,\sigma_p}$), α, and δ

Elasticity = 0.5		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.09	0.13	0.17	0.21	0.27	0.3
	0.6	0.11	0.16	0.21	0.28	0.36	0.5
	0.7	0.13	0.19	0.27	0.36	0.48	0.6
	0.8	0.16	0.24	0.33	0.46	0.64	0.9
	0.9	0.18	0.28	0.409	0.59	0.85	1.3
	0.99	0.21	0.33	0.490	0.73	1.13	1.9

Elasticity = 0.8		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.14	0.20	0.27	0.34	0.43	0.5
	0.6	0.18	0.25	0.34	0.45	0.58	0.7
	0.7	0.21	0.31	0.43	0.58	0.77	1.0
	0.8	0.25	0.38	0.53	0.74	1.02	1.4
	0.9	0.30	0.45	0.655	0.94	1.36	2.1
	0.99	0.34	0.52	0.784	1.17	1.81	3.0

Elasticity = 1		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.18	0.25	0.33	0.43	0.54	0.7
	0.6	0.22	0.32	0.43	0.56	0.72	0.9
	0.7	0.27	0.39	0.54	0.72	0.96	1.3
	0.8	0.32	0.47	0.67	0.92	1.27	1.8
	0.9	0.37	0.56	0.818	1.17	1.70	2.6
	0.99	0.42	0.66	0.980	1.46	2.26	3.8

Elasticity = 1.5		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.26	0.38	0.50	0.64	0.81	1.0
	0.6	0.33	0.47	0.64	0.84	1.09	1.4
	0.7	0.40	0.58	0.81	1.09	1.44	1.9
	0.8	0.47	0.71	1.00	1.38	1.91	2.7
	0.9	0.55	0.84	1.227	1.76	2.55	3.9
	0.99	0.63	0.98	1.470	2.19	3.39	5.7

Elasticity = 2		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.35	0.50	0.67	0.86	1.08	1.3
	0.6	0.44	0.63	0.86	1.13	1.45	1.8
	0.7	0.53	0.78	1.08	1.45	1.92	2.5
	0.8	0.63	0.94	1.33	1.85	2.55	3.6
	0.9	0.74	1.13	1.636	2.35	3.41	5.1
	0.99	0.84	1.31	1.960	2.93	4.51	7.6

Elasticity = 2.5		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.44	0.63	0.83	1.07	1.35	1.7
	0.6	0.55	0.79	1.07	1.41	1.81	2.3
	0.7	0.66	0.97	1.35	1.81	2.40	3.2
	0.8	0.79	1.18	1.67	2.31	3.18	4.4
	0.9	0.92	1.41	2.045	2.93	4.26	6.4
	0.99	1.06	1.64	2.450	3.66	5.64	9.5

Elasticity = 3		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.53	0.75	1.00	1.29	1.62	2.0
	0.6	0.66	0.95	1.29	1.69	2.17	2.8
	0.7	0.80	1.17	1.62	2.17	2.88	3.8
	0.8	0.95	1.41	2.00	2.77	3.82	5.3
	0.9	1.11	1.69	2.455	3.52	5.11	7.7
	0.99	1.27	1.97	2.941	4.39	6.77	11.4

Elasticity = 4		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.71	1.00	1.33	1.71	2.15	2.7
	0.6	0.88	1.26	1.71	2.25	2.90	3.7
	0.7	1.06	1.56	2.15	2.90	3.84	5.1
	0.8	1.26	1.88	2.67	3.69	5.09	7.1
	0.9	1.48	2.25	3.273	4.70	6.81	10.3
	0.99	1.69	2.62	3.921	5.85	9.03	15.2

Elasticity = 5		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	0.88	1.25	1.67	2.14	2.69	3.3
	0.6	1.10	1.58	2.14	2.81	3.62	4.6
	0.7	1.33	1.94	2.69	3.62	4.80	6.4
	0.8	1.58	2.35	3.33	4.62	6.36	8.9
	0.9	1.85	2.81	4.091	5.87	8.51	12.9
	0.99	2.11	3.28	4.901	7.32	11.29	19.0

Elasticity = 5.7		(highlighted values greater or equal to 1)					
$\xi_{\varepsilon_V\sigma_p}\left(\frac{\alpha.\delta}{1-\alpha.\delta}\right)$		δ = share of capital in total income					
		0.3	0.4	0.5	0.6	0.7	0.8
α = curvature of the investment's production function	0.5	1.01	1.43	1.90	2.44	3.07	3.8
	0.6	1.25	1.80	2.44	3.21	4.13	5.3
	0.7	1.52	2.22	3.07	4.13	5.48	7.3
	0.8	1.80	2.68	3.80	5.26	7.25	10.1
	0.9	2.11	3.21	4.664	6.69	9.71	14.7
	0.99	2.41	3.74	5.587	8.34	12.87	21.7