The Welfare Cost of Real Volatility: A Comparative Analysis

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Abstract

In an influential set of lectures, Lucas argues that the welfare cost associated with the business cycle in the US economy has been negligible in the post-WWII era. As a corollary of his finding, more countercyclical policies, than those already applied in US, would be unnecessary. Although most developing countries register higher macroeconomic volatility in terms of GDP and consumption per capita, there are few studies that estimate and compare the welfare consequences of macroeconomic (real) fluctuations in these economies. In this paper, I resume the discussion of how important is the welfare effect of reducing, or eliminating, the macroeconomic volatility. Staring with the Lucas framework, I provide a quantitative assessment of the welfare costs associated with real fluctuations not only for the developed countries, but also for Latin America, as a representative developing region. Two alternative models are evaluated across countries: the first one allows for a stochastic trend in the consumption process, while the second one is a general equilibrium model with uninsurable idiosyncratic labor risk. As a result, I find non-negligible welfare costs of real volatility, of more than two-order of magnitude higher than those estimated with the Lucas model.

JEL Classification: D52, E32, O40

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1 Introduction

In a seminal work, Lucas (1987, 2003) argues that the business cycle in the US economy after the WWII has generated negligible welfare cost, i.e. equivalent to less than one-hundredth of one percent of annual per capita consumption. Thus, he suggests that additional countercyclical policies beyond those already applied would be unnecessary. Since most of the related literature estimates the welfare cost generated by the US business cycle, in this article I extend the study not only to all developed countries, but also to the Latin American economies. I cover the period from 1960 to 2006, before the international financial crisis, to make my results comparable with other works previous to the crisis. I assess the welfare cost through different specifications, taking the representative agent model suggested by Lucas (1987) as the baseline case. In addition, I consider two alternative frameworks, which do not intent to match the consumption process, but to compare the welfare cost through other specifications. The first one is an extension of the baseline case by allowing for a stochastic trend in the consumption process, while the second one is a general equilibrium analysis with uninsurable idiosyncratic human capital risk following Krebs (2003). With the latter model, I find non-negligible welfare costs of more than two-order of magnitude higher than those estimated with the Lucas model. In fact, the annual increase of per capita consumption that would compensate households for living with the real volatility can reach 9.1 percent in Latin America and 7.7 percent in the US economy, compared to 0.06 percent and 0.006 percent respectively estimated with the baseline model.

The Lucas results have been tested using different assumptions and specifications. Obstfeld (1994), Dolmas (1998), and Tallarini (2000) assume Epstein-Zin preferences which better fit asset prices data by separating the elasticity of intertemporal substitution from the coefficient of relative risk aversion. This alternative specification suggests that consumption volatility is not very costly unless fluctuations are highly persistent: the welfare cost in terms of per capita consumption ranges between 0.01 percent and a maximum value of 12.6 percent, with serially correlated consumption fluctuations and higher risk-aversion.

On the other hand, there are several studies that consider consumer heterogeneity. Thus, volatility faced by some fraction of the population might be underestimated. Based on the empirical evidence, Imrohoroğlu (1989) finds that stabilization affects earning risk by avoiding long periods of unemployment, given the fact that unemployment spells are short in "good times" but longer during a recession. As a result, she estimates a cost of business cycle in the US economy of about 0.3 percent of household consumption, ruling out the interest rate risk. Krusell and Smith (1999) follow the idea that stabilization reduces the period of unemployment, but they allow for the interest rate to vary over the cycle and introduce an asymmetric wealth distribution. Under this framework, the cost of aggregate fluctuations would be 3.68 percent for those individuals who are unemployed, but smaller and even negative for households with savings. As a result, in the Krusell-Smith's framework the real volatility might be on average beneficial, depending on the fraction of the population who are unemployed and with borrowing constraints. Thus, in countries with better wealth distribution and extended access to the financial markets, stabilization policies could make the majority of people worse off. However, this is not the case of most emerging economies, which are characterized by an unequal wealth distribution and a less-developed financial system.

By looking at the US reports of household earnings, Storesletten et al. (2001) find that income shocks are highly persistent. When an idiosyncratic labor income risk is introduced and household's income falls, the earnings will be low for a longer time than in Krusell-Smith's specification. They estimate a gain from eliminating the aggregate fluctuations in the US economy of 2.5 percent of life-time consumption as a whole, while for those household without any savings the gain reach 7.4 percent. On the other hand, Krebs (2003) extends Storesletten et al. (2001) by adding permanent idiosyncratic income shocks that households are not able to offset. The welfare cost of real volatility in his calibration for the US economy is 7.4 percent of the life-time household consumption.

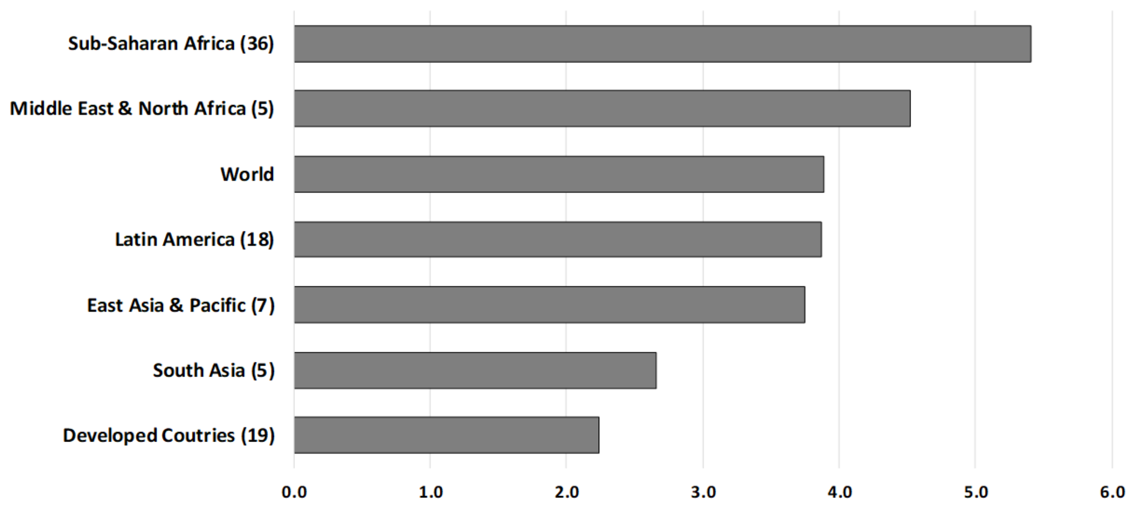
Other papers suggest that the average consumption level might change in response to stabilization policies. Instead of reverting to its trend, as in Lucas model, consumption might increase with stabilization. In other words, the consumption path in a stable economy might exceed the expected level with high macroeconomic fluctuations. Following this assumption, De Long et al. (1988) estimate a cost between 1.6 percent and 1.9 percent of US household consumption. Finally, stabilization might affect the consumption growth rates instead of the levels. Indeed, Barlevy (2004) shows that the welfare effects of real volatility might be substantial, reaching 8 percent of life-time consumption. Alternatively, Pallage and Robe (2003) is one of the few studies addressing the welfare cost of consumption volatility in poor countries. They find that the potential welfare gain from eliminating the macro volatility in poor countries exceeds the gain from one percent increase in the growth path.

In the next section, I characterize the macroeconomic fluctuations in both developing and developed countries. In the following section, the welfare costs associated with real volatility are studied through the different specifications, taking the Lucas model as the baseline case.

2 The Macroeconomic Volatility in Perspective

The standard deviation of per capita GDP growth rates is the most common indicator of macroeconomic fluctuations to capture the volatility of the whole economy. However, the second moment is also used to study the behavior of different GDP components, such as consumption, investments, exports, and so on. In general, the "volatility" of a certain variable refers to its deviation from a reference value, which gives the idea of a permanent state or trend. Since most economic variables (in levels) are non-stationary, i.e. the fluctuation is around a non-constant trend, different statistical techniques can be used to separate the permanent (trend) and transitory components (cycle).

Figure 1: Standard Deviation of per Capita GDP Growth, 1961-2016 (median by region)

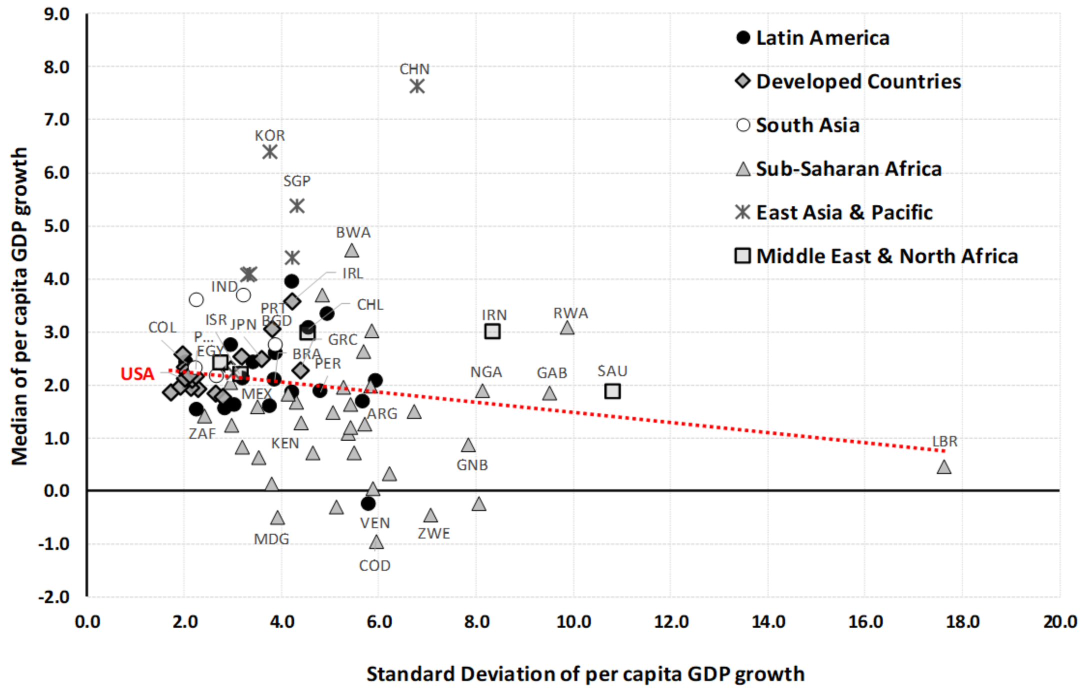


Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: GDP per capita is at constant 2010 dollars. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Although macroeconomic volatility is a worldwide phenomenon, most developing countries have registered the highest levels of GDP per capita volatility over the last five decades (Figure 1). At a country level, the real volatility is not only higher in most developing countries but it seems to have a negative effect on growth (Figure 2).This phenomenon was first documented by Ramey and Ramey (1995) and motivated an extensive literature on the negative relationship between volatility and growth.[1] Figure 2 shows that most developed countries are grouped with relatively low level of macroeconomic fluctuations and growth, while the developing countries have different patters. Therefore, the study of the welfare effects of real volatility should not only focus in the US economy, but also in the developing countries.

[1]Among others Fatás (2002), Acemoglu et al. (2003), and Hnatkovska and Loayza (2005)

Figure 2: Volatility and Growth, 1961-2016

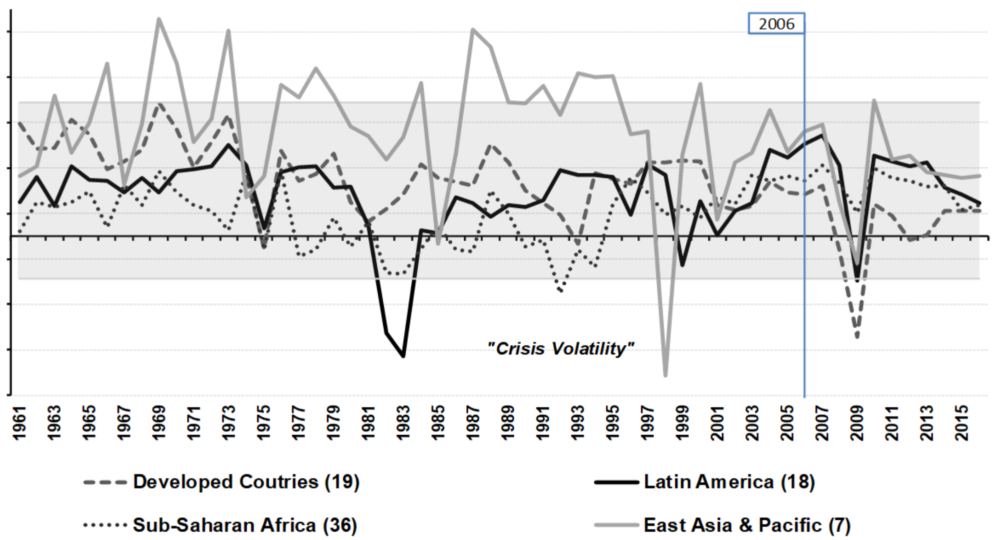


Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: GDP per capita is at constant 2010 dollars.

Hnatkovska and Loayza (2005) use an ad-hoc definition of "crisis" volatility, specifically when the per capita GDP growth lies below one standard deviation of the worldwide average of per capita GPD growth. Following this definition from 1961 to 2006, only the developing regions (i.e. Latin America, East Asia, and Sub-Saharan Africa) registered "crisis" levels of per capita GDP growth (Figure 3). However, the US economy and other developed countries became the epicenter for the international financial crisis of 2008-2009, that pulled the rest of world. The year 2006 will be the end of the sample period (1960-2006) for which the welfare cost of macroeconomic (real) volatility will be estimated in the following section. As was mentioned before, the financial crisis could easily be included in this assessment, but the resulting estimates would not be comparable to previous literature.

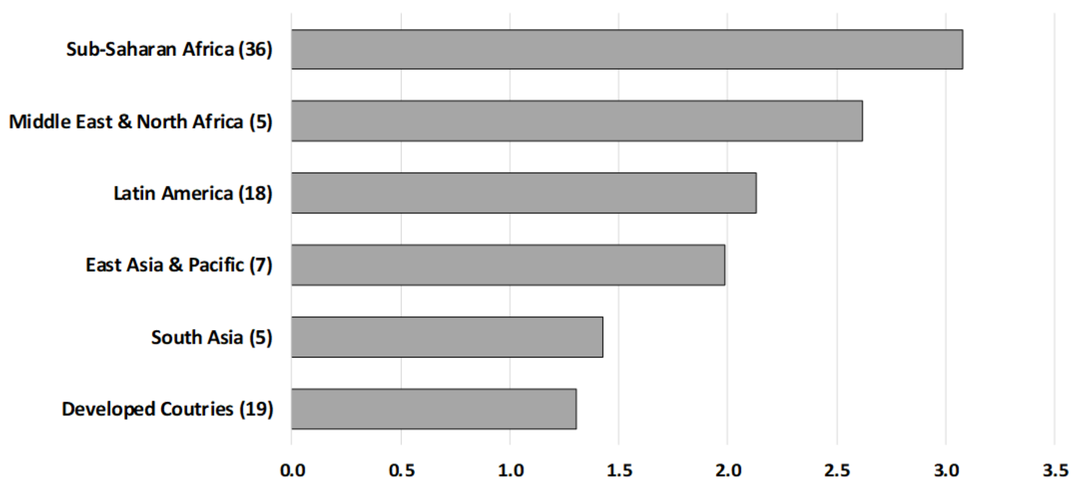
Macroeconomic volatility is also measured through the standard deviation of the cyclical component of the output per capita. Throughout this chapter the Hodrick-Prescott (HP) filter is applied to isolate the transitory components of output and consumption per capita, with a smoothing parameter $\lambda = 6.25$ (Ravn and Uhlig 2002). When looking at volatility of the business cycle for different regions, the resulted pattern replicates the ranking obtained through the standard deviation of growth rates (Figure 1). Indeed, Figure 4 shows that the developing regions lead the ranking of cycle volatility during the period.

Figure 3: Normal and Crisis Volatility, 1961-2016
(GDP per capita growth, median by region)



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: As in Hnatkovska and Loayza (2005), the shaded area defines the "normal" volatility as one standard deviation of the worldwide median of per capita GPD growth rates (in constant 2010 dollars) during the period. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Figure 4: Volatility of the Business Cycle, 1961-2016
(Percentage difference from trend, median by region)



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: The HP filter (with $\lambda = 6.25$) is applied to the natural logarithm of GDP per capita (in constant 2010 dollars) by country and the resulted cyclical component is then multiplied by 100. The number of countries in each group is given in parenthesis. The World comprises a total of 90 countries.

Although explaining the structural reasons behind the higher macroeconomic (real) volatility in most developing regions goes beyond is not the aim of this work, the literature has basically identified three main reasons to explain the persistence of high volatility on these economies:

(i) Some developing countries face larger external shocks than developed economies. For instance, in the last four decades, Latin America has suffered important external shocks, nominal and real, as a result of fluctuations in both international capital flows and terms of trade.

(ii) Production patterns are highly concentrated in more volatile industries that are intensive in non-flexible technologies and unskilled workers, whereas the structure in developed countries is based on flexible technologies and skilled workers. Kraay and Ventura (2007) argue that the cross-country differences in real volatility can be explained by different patterns in industrial specialization between developed and developing economies. Alternatively, Koren and Tenreyro (2007) show that poorer countries are specialized in fewer and more volatile sectors that explain higher macroeconomic fluctuations. In general, the previous findings support the idea that more flexible and resilient to shock technologies are chosen as the economic development increases.

(iii) Finally, institutional instabilities and inconsistent macroeconomic policies have also played an important role in increasing the real volatility in several developing countries. Macroeconomic policies have been characterized by abrupt changes in rules with deep socio-economic consequences. Most developing regions have weaker "shock absorbers" or filtering mechanisms. They have had inefficient market filters as a result of a shallow financial systems, insufficient export diversification, and endogenous rigidities associated with the political economy. Acemoglu et al. (2003) argue that most of these problems are associated with a weak institutional framework.

Given the fact that macroeconomic volatility is relatively higher in developing countries, in the next section, I analyze the welfare cost of real fluctuations through three different frameworks, not only for the US economy and other developed countries, but also for Latin American economies as a representative developing region. Since this work focuses on the welfare effect of macroeconomic fluctuations, what ultimately matters is the household consumption volatility. In a complete market scheme, households could diversify portfolios and risk, shielding its consumption from income volatility. In this context, households are able to avoid the "non-permanent" income volatility through the precautionary savings and insurance mechanisms. However, incomplete markets provide a limited protection in the context of permanent shocks. The *excess volatility of consumption* is a well known feature that characterizes many small-open economies. Indeed, Aguiar and Gopinath (2007) find that consumption is 40 percent more volatile than GDP in emerging economies, while slightly less volatility than income for developed countries.

3 Measuring the Welfare Cost of Real Volatility

The "welfare" is a subjective idea which not only depends on individual priorities but also on the pleasure or happiness that consumption generates. For this reason, the utility-function approach seems to be a natural way to measure satisfaction and to rank different consumption plans, assuming rationality and completeness. In order to measure the welfare effects of macroeconomic fluctuation, the main question throughout this section is how large the increase in annual household consumption is when the total volatility is eliminated. Formally,

$$U\left[\{(1+\Delta)\,C_t\}\right] = U\left[\{C_t^*\}\right], \tag{1}$$

where actual household consumption (C_t) deviates from the trend (C_t^*) by an assumed random variable. Thus, the cost of real volatility is represented by the fraction Δ, which makes the consumer indifferent to the economic fluctuations.

3.1 The Baseline (Lucas) Model

Following Lucas (1987), the one-period preferences are defined by a constant relative risk aversion utility function over the consumption plan:

$$U\left[\{C_t\}\right] = E_0\left[\sum_{t=1}^{\infty}\beta^t\frac{C_t^{1-\gamma}-1}{1-\gamma}\right], \text{with } 0<\beta<1 \text{ and } \gamma\geq 0. \tag{2}$$

As usual β is the rate at which utility is discounted and γ is equal to the coefficient of relative risk-aversion, so the higher γ the more reluctant is the consumer to face a volatile consumption path. Given an initial value C_0, the per capita consumption function is defined as follows:

$$C_t = (1+g)^t\,C_0e^{\varepsilon_t-\sigma_\varepsilon^2/2}, \tag{3}$$

where g is the consumption growth rate and $\varepsilon(0,\sigma_\varepsilon)$ is an i.i.d. normal random variable. In this framework, the consumption level in the economy without real fluctuations (i.e. $\sigma_\varepsilon^2=0$) is equal to the expected value of C_t. After some algebra, the percentage increase across all dates in annual consumption from eliminating the consumption uncertainty is given by

$$U\left[\{(1+\Delta)\,C_t\}\right] = \begin{cases} (1+\Delta)^{1-\gamma}\, e^{-\frac{\sigma_\varepsilon^2}{2}\gamma(1-\gamma)} = 1 & \text{if } \gamma \neq 1 \\ ln\,(1+\Delta) = \dfrac{\sigma_\varepsilon^2}{2} & \text{if } \gamma = 1. \end{cases} \quad (4)$$

Thus, the equivalent variation as a function of the consumption variance is

$$\Delta(\sigma_\varepsilon^2, \gamma) = e^{\frac{\sigma_\varepsilon^2}{2}\gamma} - 1, \;\; \forall\gamma. \quad (5)$$

Therefore, the first-order Taylor approximation of Equation (5) in the neighborhood of $\sigma_\varepsilon^2 = 0$ yields:

$$\Delta(\sigma_\varepsilon^2, \gamma) \approx \frac{\sigma_\varepsilon^2}{2}\gamma,;\; \forall\gamma. \quad (6)$$

This represents Lucas's approximation of the cost of consumption volatility, which depends on the degree of risk aversion and the variance of the shock. Therefore, real volatility is more costly the more volatile the economy becomes and the more averse the individual is. However, by using US data for consumption volatility after the WWII and assuming a log utility ($\gamma = 1$), Lucas (1987) shows that the cost of consumption fluctuation is negligible. In other words, US consumers would be willing to give up less than 0.01 percent of their consumption, uniform across all dates, in order to get macro stability. In light of these results, Lucas concludes that more aggressive countercyclical policies than those applied in the post-WWII period are not needed, since they would bring almost no benefit in terms of welfare. Before going on with the analysis it is important to clarify the scope of this finding.

Lucas does not argue that countercyclical policies are unnecessary at all from the welfare point of view. On the contrary, what he points out is that *further countercyclical policies* than those already applied in the US since 1945 would have a negligible welfare gain for consumers. Real consumption volatility would have been larger if stabilization policies had not been applied at all in US during the post-WWII. Indeed, monetary and fiscal policies have played an important role in stabilizing the US economy in the last decades, the question is whether *additional countercyclical policies would be useful.* Since the welfare effects of real volatility are analyzed in both developed and developing countries, Table 1 summarizes the welfare cost of consumption volatility using the Lucas approximation, i.e. Equation (6), not only for the United States and other developed economies, but also for Latin American countries, that represent the developing world.

In general, under the Lucas framework and reasonable values of risk aversion, eliminating the real volatility would entail negligible welfare cost across the countries. Although the cost of real fluctuations in Latin America is on average six times higher than the average cost in developed countries (ten times higher than in US), it is still lower than 0.6 percent of per capita consumption. It is straightforward to show that these findings are due to the higher consumption volatility in Latin American economies, where the standard deviation of the cyclical component of per capita consumption is on average more than two times higher than that in developed countries (see column 2).

Table 1: Welfare Effects of Consumption Volatility with the Lucas Model
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

	STDEV of the cyclical component	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	**1.1**	**0.006**	**0.009**	**0.011**	**0.028**	**0.057**
Latin America (simple average)	**3.2**	**0.059**	**0.088**	**0.118**	**0.294**	**0.588**
Argentina	**4.3**	0.094	0.142	0.189	0.472	0.945
Bolivia	**1.7**	0.015	0.022	0.030	0.074	0.148
Brazil	**2.8**	0.040	0.061	0.081	0.202	0.404
Chile	**4.8**	0.114	0.170	0.227	0.568	1.136
Colombia	**1.4**	0.010	0.015	0.020	0.050	0.100
Costa Rica	**3.1**	0.047	0.071	0.095	0.237	0.474
Ecuador	**1.5**	0.011	0.017	0.022	0.055	0.110
El Salvador	**3.5**	0.061	0.092	0.123	0.307	0.613
Guatemala	**1.0**	0.005	0.007	0.010	0.025	0.049
Honduras	**2.1**	0.022	0.033	0.044	0.110	0.219
Mexico	**2.3**	0.026	0.039	0.052	0.130	0.260
Nicaragua	**5.8**	0.171	0.256	0.341	0.853	1.707
Panama	**4.4**	0.097	0.145	0.194	0.484	0.969
Paraguay	**2.5**	0.031	0.046	0.061	0.154	0.307
Peru	**3.8**	0.073	0.110	0.146	0.366	0.731
Dominican Republic	**4.2**	0.087	0.131	0.174	0.435	0.870
Uruguay	**4.5**	0.101	0.151	0.202	0.505	1.009
Venezuela	**3.3**	0.053	0.079	0.106	0.264	0.528
Other Developed Countries (simple average)	**1.4**	**0.011**	**0.016**	**0.021**	**0.053**	**0.106**
Australia	**0.8**	0.003	0.005	0.007	0.016	0.033
Austria	**1.0**	0.005	0.008	0.010	0.025	0.051
Belgium	**0.9**	0.004	0.006	0.008	0.019	0.038
Finland	**1.8**	0.016	0.024	0.032	0.080	0.161
France	**0.6**	0.002	0.003	0.004	0.010	0.020
Greece	**1.4**	0.010	0.015	0.020	0.050	0.099
Ireland	**1.9**	0.018	0.028	0.037	0.092	0.184
Italy	**1.2**	0.007	0.011	0.015	0.037	0.074
Japan	**1.0**	0.005	0.008	0.011	0.027	0.053
Netherlands	**1.3**	0.009	0.014	0.018	0.045	0.091
New Zealand	**1.7**	0.015	0.022	0.029	0.074	0.147
Norway	**1.4**	0.010	0.016	0.021	0.052	0.103
Portugal	**2.6**	0.034	0.051	0.068	0.170	0.340
United Kingdom	**1.4**	0.009	0.014	0.018	0.046	0.092

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: To isolate the cyclical component of per capita consumption (in constant 2010 dollars), the HP filter is applied to the natural logarithm values with $\lambda = 6.25$.

On the other hand, instead of eliminating the macroeconomic volatility, it is possible to compute the welfare benefits of a one-percentage point increase in household consumption trend (g) in this framework by computing the following equivalent variation:

$$U\left[\left\{(1+\Delta)C(g,\sigma_{\varepsilon}^{2})\right\}\right] = U\left[\left\{C(g',\sigma_{\varepsilon}^{2})\right\}\right], \text{whith } g' > g, \tag{7}$$

where g and g' are the average variation of the trend component of per-capita consumption (in logs), with $g' = g(1.01)$. By using the expected life-time utility, the benefit of increasing the consumption trend is

$$\Delta(g,g',\beta,\gamma) = \begin{cases} \left[\dfrac{1-\beta(1+g)^{1-\gamma}}{1-\beta(1+g')^{1-\gamma}}\right]^{\frac{1}{1-\gamma}} - 1 & \text{if } \gamma \neq 1 \\ e^{\left(\frac{\beta}{1-\beta}\right) ln\left(\frac{1+g'}{1+g}\right)} - 1 & \text{if } \gamma = 1. \end{cases} \tag{8}$$

Table 2 reports the aggregate results for an increase in one-percentage point of the trend consumption growth rate (g'). Notice that the benefits rises as the discount factor (β) increases, but falls as the risk aversion (γ) increases; this is because γ negatively affects the effective discount factor applied to future consumption. The benefits are considerably higher in both developed and developing countries. The extra percent point of trend growth is equivalent to a significant increase of more than 20 percent in consumption per capita across all dates, reaching more than 37 percent for $\beta = 0.97$. As a result, the conclusion in Lucas (1987) seems to be applicable for all countries, that is, the policy makers should focus on increasing the trend growth rates, rather than reducing the macroeconomic fluctuations. In the Lucas representative agent model, the only way to get considerable welfare effects is by increasing the trend growth rate of per capita consumption instead of reducing the volatility. Nonetheless, two alternative specifications will be analyzed in the following sections.

Table 2: Benefit of One-Percent Increase of the Trend of per Capita Consumption
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

		Discount Factor (β) =0.95					Discount Factor (β) =0.97				
	g	γ=1.0	γ=1.5	γ=2	γ=5	γ=10	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	2.4	20.3	15.5	12.5	5.6	2.8	36.9	23.8	17.5	6.6	3.0
Latin America	1.5	20.5	17.0	14.5	7.9	4.6	37.3	27.3	21.8	10.1	5.5
Other Developed Countries	2.5	20.3	15.3	12.3	5.5	2.7	36.9	23.4	17.2	6.4	3.0

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: To isolate the cyclical component of per capita consumption (in constant 2010 dollars), the HP filter is applied to the natural logarithm values with $\lambda = 6.25$. Simple average by country group.

3.2 Alternative (I): *A representative agent framework with a stochastic trend in the consumption process*

Lucas (1987) assumes that consumption is generated by a trend-stationary process as in Equation (3), however, Hall (1978) suggests that the consumption process contains a unit root under the permanent income hypothesis with rational expectations. Table 3 shows the results from different unit root tests (Augmented Dickey-Fuller, ADF and the Phillips-Parron, PP) with the null hypothesis that the per capita consumption path (in natural logarithms) has a stochastic trend. The null is not rejected in almost all countries in the sample. Even though the ADF test rejects the null for the US and Netherlands, the PP test does not. Reis (2009) conducts a variety of statistical tests to investigate whether the consumption in United States has a unit root (including the Elliot-Rothenberg-Stock and Ng-Perron tests), finding that null hypothesis is never rejected at significance level of 5 percent.

Since the martingale is the simplest specification that describes a process with a unit root, let the natural logarithms of per capita consumption (c_t) follow:

$$c_t = c_0 + \left(g - \frac{1}{2}\sigma_\varepsilon^2\right) t + \sum_{i=1}^{t} \varepsilon_i. \tag{9}$$

As before g is the trend annual growth rate, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, and the initial value c_0 is $ln(C_0)$. A similar analysis is found in Obstfeld (1994), including the term $-\frac{1}{2}\sigma_\varepsilon^2$ to ensure that the increase in the variance of ε_t does not affect the mean. Although Lucas (1987) does not completely rule out the possibility of a unit root in the consumption process, he would prefer an intermediate case between (3) and (9).[2] Under (9), the level of per capita consumption function becomes

$$C_t = C_0 e^{\left(g - \frac{1}{2}\sigma_\varepsilon^2\right)t} e^{\sum_{i=1}^{t} \varepsilon_i}. \tag{10}$$

Following Obstfeld (1994), the equivalent variation to keep the agent indifferent if consumption process changes from $(C_0, g, \sigma_\varepsilon^2)$ to $(C_0, g', \sigma_\varepsilon'^2)$ in (9) is defined by

$$U\left[\left\{C((1+\Delta)C_0, g, \sigma_\varepsilon^2)\right\}\right] = U\left[\left\{C(C_0, g', \sigma_\varepsilon'^2)\right\}\right]$$

If consumption follows a martingale process, then the equivalent variations (Δ) is

$$\Delta(g, g', \sigma_\varepsilon^2, \sigma_\varepsilon'^2, \beta, \gamma) = \begin{cases} \left[\dfrac{1 - \beta e^{(1-\gamma)\left(g - \frac{\gamma\sigma_\varepsilon^2}{2}\right)}}{1 - \beta e^{(1-\gamma)\left(g' - \frac{\gamma\sigma_\varepsilon'^2}{2}\right)}}\right]^{\left(\frac{1}{1-\gamma}\right)} - 1 & \text{if } \gamma \neq 1 \\ e^{\left[g' - g - \frac{(\sigma_\varepsilon'^2 - \sigma_\varepsilon^2)}{2}\right]\frac{\beta}{1-\beta}} - 1 & \text{if } \gamma = 1. \end{cases} \tag{11}$$

To estimate the case when $g = g'$ and $\sigma_\varepsilon'^2 = 0$, that is, a change to an economy with the

[2] See Lucas (1987), pp. 22-23, footnote 1

Table 3: Unit Root Tests on Consumption per capita, 1961-2006

	Augmented Dickey Fuller Test		Phillips-Perron Test	
	Test Statistics	Decision at 5% critical value (-3.513)	Test Statistics	Decision at 5% critical value (-3.511)
Latin America				
Argentina	-3.31	NR	-2.83	NR
Bolivia	-1.97	NR	-2.07	NR
Brazil	-1.06	NR	-1.18	NR
Chile	-1.83	NR	-1.24	NR
Colombia	-1.78	NR	-1.78	NR
Costa Rica	-2.30	NR	-2.77	NR
Ecuador	-1.66	NR	-1.64	NR
El Salvador	-1.92	NR	-1.56	NR
Guatemala	-2.72	NR	-1.95	NR
Honduras	-1.95	NR	-1.95	NR
Mexico	-2.38	NR	-1.79	NR
Nicaragua	-1.44	NR	-1.70	NR
Paraguay	-0.74	NR	-1.13	NR
Peru	-3.02	NR	-2.45	NR
Uruguay	-2.75	NR	-2.39	NR
Venezuela	-2.06	NR	-1.50	NR

	Augmented Dickey Fuller Test		Phillips-Perron Test	
	Test Statistics	Decision at 5% critical value (-3.513)	Test Statistics	Decision at 5% critical value (-3.511)
Developed Countries				
United States	-3.66	R	-2.26	NR
Australia	-1.71	NR	-1.79	NR
Austria	-0.74	NR	-0.46	NR
Belgium	-0.48	NR	-0.72	NR
Denmark	-3.49	NR	-3.49	NR
Finland	-1.97	NR	-2.25	NR
France	-3.16	NR	-2.76	NR
Greece	-2.27	NR	-2.25	NR
Ireland	-1.97	NR	-1.42	NR
Italy	-2.24	NR	-1.48	NR
Japan	-2.89	NR	-2.67	NR
Netherlands	-3.83	R	-2.06	NR
New Zealand	-1.96	NR	-1.22	NR
Norway	-2.06	NR	-2.20	NR
Portugal	-3.18	NR	-1.79	NR
Spain	-2.68	NR	-3.39	NR
Sweden	-2.68	NR	-2.75	NR
United Kingdom	-2.97	NR	-2.22	NR

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: ADF test includes a constant and a time trend. "NR" stands for "Not-Rejected" and "R" for "Rejected".

same consumption growth but without volatility, (11) becomes

$$\Delta(\hat{g}, \hat{\sigma}_{\varepsilon}^{2}, \beta, \gamma) = \begin{cases} \left[\dfrac{1 - \beta e^{(1-\gamma)\left(\hat{g} - \frac{\gamma \hat{\sigma}_{\varepsilon}^{2}}{2}\right)}}{1 - \beta e^{(1-\gamma)\hat{g}}} \right]^{\left(\frac{1}{1-\gamma}\right)} - 1 & \text{if } \gamma \neq 1 \\ e^{\frac{\hat{\sigma}_{\varepsilon}^{2}}{2}\left(\frac{\beta}{1-\beta}\right)} - 1 & \text{if } \gamma = 1. \end{cases} \quad (12)$$

To calibrate this alternative model, I regress the first-difference of per capita consumption in logs ($\varDelta c_t$) on a constant by ordinary least squareto obtain an estimate of standard errors ($\hat{\sigma}_{\varepsilon}$). After that, point estimates for $\hat{g}$ in specification (9) is the result of running the previous regression again, but with a constant and adding $\frac{1}{2}\hat{\sigma}_{\varepsilon}^{2}$ to the dependent variable $\varDelta c_t$.

Table 4 reports the welfare cost of consumption fluctuations estimated by (12) for different values of risk aversion (γ) and discount factors (β). The first two columns show estimate of $\hat{g}$ and $\hat{\sigma}_{\varepsilon}$.[3] By allowing for a unit root in the consumption process, the average welfare loss in developed countries (for $\gamma = 1$ and $\beta = 0.95$) is more than sixty times the average cost estimated by the baseline (Lucas) model, and it is more than forty times for Latin American countries. Even more, for $\beta = 0.97$ the average welfare effects in both developed countries and Latin America might be more than one hundred times and sixty times the baseline calculations respectively. This result is explained by the larger standard deviations

[3] Countries are not reported when $\hat{g}$ is not statistically significant at the 10 percent level.

under (9) than those calculated after applying the HP filter in the Lucas model.

Again the Latin American region registers the highest welfare cost associated with the macroeconomic volatility. While the welfare cost in US is under 0.5 percent of per capita consumption for any value of γ and β, the average cost in Latin America ranges between 2.4 percent and 11.9 percent of per capita consumption, with countries like Argentina and Chile where the estimate welfare cost exceeds the 10 percent for intermediate values of γ and β (say 5 and 0.96 respectively). These larger welfare costs suggest that further countercyclical macro policies than those applied since 1960 are needed in most of Latin American countries. However, before drawing any conclusion an alternative model will be analyzed in the next section, which incorporates idiosyncratic risk and incomplete markets.

Table 4: Welfare Effects When the Consumption Process has a Unit Root
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

	$\bar{g}$	$\bar{\sigma}_\varepsilon$	Discount Factor (β) =0.95					Discount Factor (β) =0.97				
			γ=1.0	γ=1.5	γ=2	γ=5	γ=10	γ=1.0	γ=1.5	γ=2	γ=5	γ=10
United States	**2.39** (0.00)	**1.6**	**0.23**	**0.28**	**0.31**	**0.38**	**0.40**	**0.39**	**0.42**	**0.43**	**0.45**	**0.44**
Latin America	**1.90**	**4.6**	**2.4**	**3.0**	**3.4**	**5.1**	**9.5**	**4.1**	**4.6**	**4.9**	**6.5**	**9.7**
Argentina	**1.68** (0.08)	**6.5**	4.1	5.3	6.3	10.9	33.0	7.0	8.4	9.5	14.4	-
Bolivia	**0.83** (0.07)	**3.0**	0.8	1.2	1.5	2.7	4.1	1.4	1.9	2.3	3.7	5.2
Brazil	**2.27** (0.01)	**5.2**	2.6	3.2	3.7	5.1	6.8	4.5	5.0	5.3	6.2	7.8
Chile	**2.29** (0.03)	**7.0**	4.8	5.9	6.8	10.4	22.9	8.3	9.2	9.9	13.1	34.0
Colombia	**1.75** (0.00)	**2.5**	0.6	0.8	0.9	1.3	1.5	1.0	1.2	1.3	1.6	1.7
Costa Rica	**1.77** (0.02)	**4.8**	2.2	2.8	3.3	5.0	7.1	3.7	4.4	4.8	6.3	8.4
Ecuador	**1.88** (0.00)	**2.9**	0.8	1.0	1.2	1.6	1.9	1.4	1.6	1.7	2.0	2.1
Guatemala	**1.31** (0.00)	**2.0**	0.4	0.5	0.6	0.9	1.1	0.6	0.8	0.9	1.1	1.3
Honduras	**1.26** (0.02)	**3.5**	1.2	1.6	1.9	3.2	4.4	2.0	2.5	2.9	4.1	5.3
Mexico	**2.00** (0.00)	**3.5**	1.2	1.5	1.7	2.4	2.8	2.0	2.3	2.5	2.9	3.2
Panama	**3.00** (0.01)	**7.1**	4.9	5.8	6.4	8.5	13.4	8.5	8.7	8.9	10.1	15.7
Dominican Republic	**3.13** (0.00)	**6.6**	4.2	4.9	5.3	6.8	9.2	7.3	7.3	7.3	8.0	10.4
Venezuela, RB	**1.53** (0.07)	**5.6**	3.0	4.0	4.7	8.0	15.3	5.1	6.3	7.1	10.5	21.1
Developed countries	**2.6**	**2.5**	**0.6**	**0.7**	**0.8**	**1.0**	**1.1**	**1.0**	**1.1**	**1.1**	**1.2**	**1.2**
Selected Countries:												
France	**2.48** (0.00)	**1.5**	0.23	0.27	0.30	0.37	0.38	0.38	0.41	0.42	0.43	0.42
Italy	**2.88** (0.00)	**2.4**	0.53	0.61	0.67	0.78	0.80	0.90	0.91	0.91	0.90	0.87
Japan	**3.52** (0.00)	**2.9**	0.80	0.89	0.94	1.02	1.01	1.37	1.30	1.26	1.16	1.09
United Kingdom	**2.35** (0.00)	**2.0**	0.37	0.45	0.51	0.64	0.68	0.64	0.69	0.71	0.76	0.75

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Simple average by country group.

3.3 Alternative (II): *A general equilibrium model with an uninsurable idiosyncratic labor risk*

The baseline model assumes a representative agent and complete markets where the macroeconomic volatility does not affect the economic growth. In this section, I deviate from the representative agent environment by studying a general equilibrium model following Krebs (2003) that incorporates an uninsurable idiosyncratic labor risk. Since the labor risk is uninsurable, there is no asset in the financial market with payoffs tied to the realizations of the idiosyncratic shocks. Although it is difficult to state from the data whether income shocks are permanent or just highly persistent, in this model the income shock is taken as permanent.[4]

The Economy. There is an aggregate productivity shock (S_t) that affects the returns of both physical and human capital investments. In addition, let s_t be a human capital shock that only affects the human capital accumulation. Both shocks are unpredictable, so they have an i.i.d. distribution over time. There are $i = 1, .., I$ ex-ante identical households, meaning that the idiosyncratic shocks are identically distributed across them. There is one non-perishable good that can be consumed or invested and only one firm that produces it. This firm produces Y_t units of the all-proposed good by employing physical (K_t) and human capital (H_t) with a standard neoclassical production function with constant-returns-to-scale. Formally, $Y_t = A(S_t)F(K_t, H_t)$, where A is a total factor productivity function $A\colon S \to \mathbb{R}^+$ that assigns a productivity level for each aggregate state.[5] The gross physical and human capital returns are denoted by $\hat{r}_k$ and $\hat{r}_h$ respectively. Hence, the firm faces the following static maximization problem:

$$\max_{K_t, H_t} \quad A(S_t)F(K_t, H_t) - \hat{r}_{kt}K_t - \hat{r}_{ht}H_t. \tag{13}$$

Let's define some useful variables: the capital-to-labor ratio $\tilde{k}_{it} = \frac{k_{it}}{hit}$, the total wealth $w_{it} = k_{it} + h_{it}$, and the net returns of physical and human capital as $r_{kt} = \hat{r}_{kt} - \delta_k$ and $r_{ht} = \hat{r}_{ht} - \delta_h$. For convenience, the average depreciation rates of physical and human capital (δ_k and δ_h respectively) are constant and independent of the aggregate risk.[6]

Households have identical time-additive preference over the consumption plan $\{c_{it}\}$.

[4] Imrohoroğlu (1989) and Krusell et al. (2009) consider shocks with some degree of persistence.

[5] All standard neoclassical properties hold, in particular F is twice-continuously differentiable, strictly concave and $\lim_{K\to 0} F'_K = \lim_{H\to 0} F'_H = +\infty$ and $\lim_{K\to\infty} F'_K = \lim_{H\to\infty} F'_H = 0$

[6] Krebs (2003) assumes that the depreciation rate of physical and human capital are equal and depend on the aggregate shocks, i.e. $\delta_{kt} = \delta_{ht} = \delta(S_t)$.

Consequently, each household i solves the following maximization problem:

$$\max_{\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\}} \quad \sum_{t=0}^{\infty} E\left[\beta^t u(c_{it})\right], \tag{14a}$$

$$\text{s.t.} \quad w_{it+1} = w_{it}\left[1 + \frac{\tilde{k}_{it}}{1+\tilde{k}_{it}} r_{kt} + \frac{1}{1+\tilde{k}_{it}}\left(r_{ht} + \eta_{it}\right)\right] - c_{it}, \tag{14b}$$

$$c_{it} \geq 0, w_{it+1} \geq 0, k_{it+1} \geq 0, \forall t, \text{ where } w_{i0} \text{ and } \tilde{k}_{i0} \text{ are given.} \tag{14c}$$

The term η_{it} is the household-specific shock that only affects the stock of human capital with $E\left(\eta_{it}|S_t\right) = 0$. The function $\eta\colon s \times S \to \mathbb{R}$ assigns to each pair (s, S) a realization η. The idiosyncratic shock is taken as another source of human capital depreciation, and is related to the specific-skill losses or skills no longer used in the event of a job displacement. Consequently, there is some part of the accumulated human capital that is either destroyed or made obsolete when an agent loses his job, in that case $\eta_{it} < 0$. Of course, this cost is more severe in a context of an economic downturn, where the aggregate risk affects the variance of the idiosyncratic shock. This model focuses on the permanent wage loss instead of taking into account the missed wage during the unemployment period.

The Equilibrium.The recursive equilibrium can be found by solving a decision problem of a household who lives in autarky and faces the same problem every period with different sets of information. Focusing on the stationary recursive equilibria with a constant physical-to-human capital ratio, $\tilde{K}_t = \tilde{K} = K_t/H_t$, the equilibrium will be the sequences of prices and actions,$\{r_{kt}, r_{ht}\}$, $\{k_t, h_t\}$ and $\left\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\right\}$, where:

(i) $\{k_t, h_t\}$ solves the firm maximization problem (13).

(ii) $\left\{c_{it}, w_{it+1}, \tilde{k}_{it+1}\right\}$ maximizes (14a) subject to the sequential budget constraint (14b).

(iii) Markets clear.

The FOCs associated with the firm's maximization problem gives the return of physical and human capital

$$r_k(\tilde{K}, S_t) = A(S_t)F'_K - \delta_k, \tag{15a}$$

$$r_h(\tilde{K}, S_t) = A(S_t)F(\tilde{K}, 1) - \tilde{K}F'_K - \delta_h. \tag{15b}$$

However, the market clearing condition implies that the aggregate capital-labor ratio must equal to the firm's capital-to-labor ratio

$$\frac{\sum_i^I \dfrac{\tilde{k}_{it}}{1+\tilde{k}_{it}} w_{it}}{\sum_i^I \dfrac{1}{1+\tilde{k}_{it}} w_{it}} = \frac{\sum_i^I k_{it}}{\sum_i^I h_{it}} = \frac{K_t}{H_t} = \tilde{K}. \tag{16}$$

If the households's capital-labor ratios are symmetric, $\tilde{k}_{it} = \tilde{k}$, then (16) is satisfied if only if $\tilde{K} = \tilde{k}$. Consequently, the returns defined in (15a) and (15b) become $r_k(\tilde{k}, S_t)$ and $r_h(\tilde{k}, S_t)$ respectively. Assuming a constant degree of relative risk aversion, the one-period utility function is $u(c_{it}) = c_{it}^{1-\gamma}/(1-\gamma)$ if $\gamma \neq 1$, or $u(c_{it}) = ln(c_{it})$ if $\gamma = 1$. Thus, the two Euler equations associated with the maximization problem (14) for a constant and symmetric capital-labor ratio are as follows

$$c_{it}^{-\gamma} = \beta E_t \left\{ c_{it+1}^{-\gamma} \left[1 + \frac{\tilde{k}}{1+\tilde{k}} r_k(\tilde{k}, S_{t+1}) + \frac{1}{1+\tilde{k}} \Big(r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) \Big) \right] \right\}, \tag{17}$$

and

$$E_t \left\{ c_{it+1}^{-\gamma} \left[r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) - r_k(\tilde{k}, S_{t+1}) \right] \right\} = 0. \tag{18}$$

The first Euler condition basically states that the marginal utility loss (utility cost) of investing (saving) one more unit of good must be equal to the expected utility gain of doing so. The second Euler equation states the equality of expected returns on the two investment opportunities (marginal utility weighted). As noted by Krebs (2003), the solution exits if

$$\sup_{\tilde{k}} \beta E \left[\left(1 + \frac{\tilde{k}}{1+\tilde{k}} r_k + \frac{1}{1+\tilde{k}} (r_h + \eta_i) \right)^{1-\gamma} \right] < 1. \tag{19}$$

Direct calculation shows that the plan $c_{it} = a(1+r_{it})w_{it}$ solves the Euler equations together with the budget constraint. Defining $w_{it+1} = (1-a)(1+r_{it})w_{it}$, the Euler conditions (17) and (18) become

$$a = 1 - \left[\beta E_t \left[(1+r_{t+1})^{1-\gamma} \right] \right]^{1/\gamma}, \text{if } \gamma \neq 1, \text{ or } a = 1-\beta, \text{ if } \gamma = 1, \tag{20a}$$

$$E_t \left[\frac{r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) - r_k(\tilde{k}, S_{t+1})}{(1+r_{t+1})^{\gamma}} \right] = 0. \tag{20b}$$

Condition (19) ensures that $0 < a < 1$. Additionally, (20a) and (20b) define not only the equilibrium values of a and $\tilde{k}$, but also that transversality condition holds, $\lim_{t\to\infty} \beta^t E_t \left[c_{it}^{-\gamma} w_{it+1} \right] \to 0$, so in this case the Euler equations are necessary and sufficient. Therefore, the allocations of the simple recursive equilibrium are given by

$$\begin{aligned} \tilde{k}_{it} &= \tilde{k}; & k_{it} &= \frac{\tilde{k}}{1+\tilde{k}} w_{it}; \\ c_{it} &= a\left[1 + r(\tilde{k}, s_{it}, S_t)\right] w_{it}; & & \\ w_{it+1} &= (1-a)\left[1 + r(\tilde{k}, s_{it}, S_t)\right] w_{it}; & h_{it} &= \frac{1}{1+\tilde{k}} w_{it}; \end{aligned} \tag{21}$$

where $r(\tilde{k}, s_{it}, S_t) = \frac{\tilde{k}}{1+\tilde{k}} r_{kt} + \frac{1}{1+\tilde{k}}(r_{ht} + \eta_{it})$ is the total return on investments and a is the consumption-to-wealth ratio. Finally, the aggregate returns are

$$r_{kt} = r_k(\tilde{k}, S_t) \quad \text{and} \quad r_{ht} = r_h(\tilde{k}, S_t). \tag{22}$$

Model Calibration. Following Krebs (2003), a Cobb-Douglas production function with constant returns-to-scale is considered, $f(\tilde{k}_t) = A_t \tilde{k}_t^{\alpha}$, as well as logarithmic preferences ($\gamma = 1$). Thus, (20a) and (20b) read

$$a = 1 - \beta, \tag{23a}$$

$$E_t\left[\frac{r_h(\tilde{k}, S_{t+1}) + \eta(s_{it+1}, S_{t+1}) - r_k(\tilde{k}, S_{t+1})}{(1 + r_{t+1})}\right] = 0. \tag{23b}$$

The aggregate per capita consumption growth rate in this economy is obtained by the individual (gross) consumption growth in expression (21) and the law of large numbers

$$\mu(S) = \frac{C_{t+1}}{C_t} = E\left[\frac{c_{it+1}}{c_{it}} | S_{t+1}\right] = (1-a)\left[1 + E\left[r(\tilde{k}, s_{it+1}, S_{t+1})\right]\right]. \tag{24}$$

Then, $\mu(S)$ is an i.i.d. process.[7] Considering the value of a and the assumption of a Cobb-Douglas production function, the per-capita consumption growth (24) become

$$\mu(S) = \beta\left[1 + \frac{\tilde{k}}{1+\tilde{k}} A_t \alpha \tilde{k}^{\alpha-1} + \frac{1}{1+\tilde{k}} A_t (1-\alpha) \tilde{k}^{\alpha} - \tilde{\delta}\right], \tag{25}$$

where $\tilde{\delta}$ is the total depreciation rate of both physical and human capital. The quantitative analysis is based on annual data (1960-2006) for developed and Latin American countries, with parameter values extensively used in RBC models. The share of capital in total income (α) is equal to 0.36.[8] Furthermore, the common discount factor (β) equals 0.96 to ensure that all households have the same intertemporal preferences.[9] As in Krebs (2003) and

[7]See Campbell and Cochrane (1999)

[8]Although this value is commonly used in the calibration of RBC models, it might not match the real income share in Latin American countries.

[9]This β value is suggested by Cooley and Prescott (1995). Even though it is possible to derivate β from (25), this would stand for different preferences in each country with unrealistic values for some countries.

Jones et al. (1999), the total depreciation rate $\tilde{\delta}$ is set in 0.06. If a common choice for the depreciation rate of physical capital is $\delta_k = 0.05$ (see Cooley and Prescott (1995)), then the depreciation rate of human capital would be $\delta_h = 0.01$.

I assume two-state shocks in the aggregate economy: the low level (L) and a high level (H) of the economic cycle. Thus, S has two possible realizations $\{L, H\}$ with equal probabilities $\pi(L) = \pi(H) = 0.5$. In addition, the human capital shock (s_{it}) follows a standard normal distribution, which is independent of the aggregate shock S. The household-loss of human capital is defined as follows

$$\eta(s_i, S) = \sigma_\eta(S) s_i, \tag{26}$$

therefore $\eta_{it} \sim \mathcal{N}(0, \sigma_\eta^2(S_t))$. The *integration principle*, which was first proposed by Krusell and Smith (1999), is applied to eliminate the aggregate risk (business cycle).[10] Thus, the function (26) is replaced by

$$\eta^*(s_i) = 0.5 \left[\sigma_\eta(L) + \sigma_\eta(H)\right] s_i. \tag{27}$$

The idiosyncratic human capital risk is transformed from the heteroscedastic to homoscedastic process: $\eta_{it} \sim \mathcal{N}(0, \sigma_\eta^2(S_t))$ to $\eta_{it}^* \sim \mathcal{N}\left(0, E\left[\sigma_\eta^2(S_t)\right]\right)$, where $E\left[\sigma_\eta^2(S_t)\right] = 0.5\left[\sigma_\eta^2(L) + \sigma_\eta^2(H)\right]$.

The expected value of per-capita consumption growth for each economy matches the per-capita household consumption data in each country for the period 1960-2006, obtaining the total factor productivity for the two states: $A(L)$ and $A(H)$. In order to take into account the consumption volatility, the growth rates for the two possible states depend on the standard deviation of the per capita consumption growth rate in each country:

$$\begin{aligned} &E\left[g(S_t)\right] = 0.5\left[\mu(L) + \mu(H)\right], \\ &\text{with } \mu(L) = \mu - \sigma_\mu, \text{ and } \mu(L) = \mu - \sigma_\mu. \end{aligned} \tag{28}$$

Next, I calculate the equivalent (or adjusted) average growth rate ($\hat{\mu}$) of the per capita consumption, which is defined as the certain growth rate for which expected life-time utility is equal to that associated with the uncertain growth rate $g(S) \sim \mathcal{N}(\mu(S), \sigma_g(S))$ with $\sigma_g(S) = \beta\sigma_y(S)$ and keeping the initial consumption constant:

[10] Also see Krusell et al. (2009)

$$
\begin{aligned}
E\left[\sum_{t=0}^{\infty}\beta^t log(c_{it})\right] &= \frac{1}{1-\beta}log(c_{i0}) + \frac{\beta}{(1-\beta)^2}E\left[log(1+g(S))\right] \\
&= \frac{1}{1-\beta}log(c_{i0}) + \frac{\beta}{(1-\beta)^2}log(1+\hat{\mu}) \\
&= E\left[\sum_{t=0}^{\infty}\beta^t log(c_{it}^*)\right], \qquad (29)
\end{aligned}
$$

where $c_{it}^* = c_{i0}(1+\hat{\mu}(S))^t$. Solving for $\hat{\mu}(S)$ yields

$$
\hat{\mu}(S) = e^{E[log(1+g(S))]} - 1. \qquad (30)
$$

The risk-adjusted consumption growth rate is independent of the initial level (c_{i0}) and is also equal for all households and the aggregate economy. The expression $E[log(1+g(S)]$ is calculated through the Gauss-Hermite quadrature rule for expectations of functions of a normal random variable with two nodes.[11] The per capita consumption growth rate changes to a homoscedastic process, i.e. $g' \sim \mathcal{N}(E[\mu(S)], E[\sigma_g(S)])$, when the aggregate risk is eliminated. Then, the change in the risk-adjusted growth rate can be estimated by

$$
\Delta\hat{\mu} = \hat{\mu}' - E[\hat{\mu}(S)] = e^{E[log(1+g')]} - 0.5\left[\hat{\mu}(L) + \hat{\mu}(H)\right]. \qquad (31)
$$

This expression approximates the welfare change expressed in risk-adjusted growth rates. However, to compare this calibration procedure with the baseline and first-alternative models, I need to get the equivalent change in consumption levels that has the same positive effect of eliminating the business cycle. By direct calculation

$$
\begin{aligned}
& log(c_{i0}(1+\Delta c)) + \frac{\beta}{1-\beta}E\left[log(1+g(S))\right] = log(c_{i0}) + \frac{\beta}{1-\beta}E\left[log(1+g')\right], \\
& \Delta c = e^{\frac{\beta}{1-\beta}\{E[log(1+g')] - E[log(1+g(S))]\}} - 1, \\
& \Delta c \approx \frac{\beta}{1-\beta}\Delta\hat{\mu}. \qquad (32)
\end{aligned}
$$

Given the expression of individual labor income ($y_{iht} = (r_h + \delta_h)h_{it}$) and (21), the law of motion of human capital is

$$
h_{it+1} = (1-\theta)w_{it+1} = \beta\left[1 + \theta r_k + (1-\theta)(r_h + \eta_{it})\right]h_{it}, \qquad (33)
$$

where $\theta = \tilde{k}/(1+\tilde{k})$ is the total wealth invested in physical capital. To follow the empirical

[11] Algebraically: $E[log(1+g(S)] \approx \pi^{-1/2}\sum_{i=1}^{2}\omega_i log\left[1 + \left(\sqrt{2}\sigma_g(S)x_i + \mu(S)\right)\right]$, where x_i are the quadrature-nodes with weights ω_i. See Judd (1998) for nodes values and weights.

literature, the first difference of the labor income in logs approximately follows a random walk with drift:[12]

$$\begin{aligned} log(y_{it+1}) - log(y_{it}) &= log\left[(r_h+\delta_h)h_{it+1}\right] - log\left[(r_h+\delta_h)h_{it}\right] \\ &= log(h_{it+1}) - log(h_{it}) \\ &= log(\beta) + log\left[1+\theta r_k + (1-\theta)(r_h+\eta_{it})\right] \quad \text{by(33)} \\ &\approx \underbrace{log(\beta)+\theta r_k+(1-\theta)r_h}_{\textbf{Drift } (Z)} + \underbrace{(1-\theta)\eta_{it}}_{\tilde{\eta}_{it}}, \text{ since } log(1+r)\approx r, \end{aligned} \tag{34}$$

with an error term $\tilde{\eta}_{it} \sim \mathcal{N}\left(0, \sigma_y^2(S_t)\right)$, where the income variance will depend on aggregate risk (i.e. the business cycle)

$$\sigma_y^2(S_t) = (1-\theta)^2\sigma_\eta^2(S_t) = \frac{\sigma_\eta^2(S_t)}{(1+\tilde{k})^2}. \tag{35}$$

Perhaps the most difficult step in the calibration process is to find a suitable approximation of the standard deviation defined in (35), because there is not enough empirical literature that estimates the income labor with an idiosyncratic risk component for Latin American countries. As a benchmark, I take an average of values reported by Krebs (2003), based on different empirical studies that estimate the standard deviation of the error term for a random walk specification of the labor income (σ_y) in the US economy:[13]

$$\sigma\left[{}^{y_{iht+1}}/_{y_{iht}}|S_t = L\right] = 0.26 \quad \text{and} \quad \sigma\left[{}^{y_{iht+1}}/_{y_{iht}}|S_t = H\right] = 0.10. \tag{36}$$

Although the estimated labor income volatility based on the US data (mainly PSID data) might be generalized for some developed countries, it can underestimate the actual labor income volatility in Latin American countries, since these economies have higher per capita income volatility (as discussed in the Section 1.1) and an extended informal labor market.[14] Therefore, it would be reasonable to assume that the higher macroeconomic volatility and more informal workers in these economies might translate into higher labor-income volatility. As a result, if the labor income volatility based on the US data (36) is used to estimate the welfare effects for developing countries (in this case for Latin America), the results should be taken as a lower-bound estimate.

Table 5 shows the expected value of the per capita consumption growth rate ($E[\mu(S)]$), the change in the risk-adjusted growth rate ($\Delta\hat{\mu}$) from eliminating the real fluctuations, and

[12] See Krebs (2007) and Storesletten et al. (2001)

[13] For some empirical studies see Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Storesletten et al. (2007).

[14] On average the informal economy in Latin America represents 42 percent of GNP but 18 percent in the high-income OECD countries (Schneider (2002)).

the welfare effects in terms of equivalent changes in consumption level (Δc). Even though the US labor income volatility is used across the countries, the welfare costs associated with the business cycle are considerably higher than those estimated in the previous sections. Indeed, an average welfare cost of 8.0 percent of per capita consumption in developed economies is estimated, while the average welfare cost in Latin America is more than 9 percent. As was mentioned before, the latest value for Latin American countries should be taken as a lower-bound estimate since the labor income volatility in most developing countries might be higher than in US.

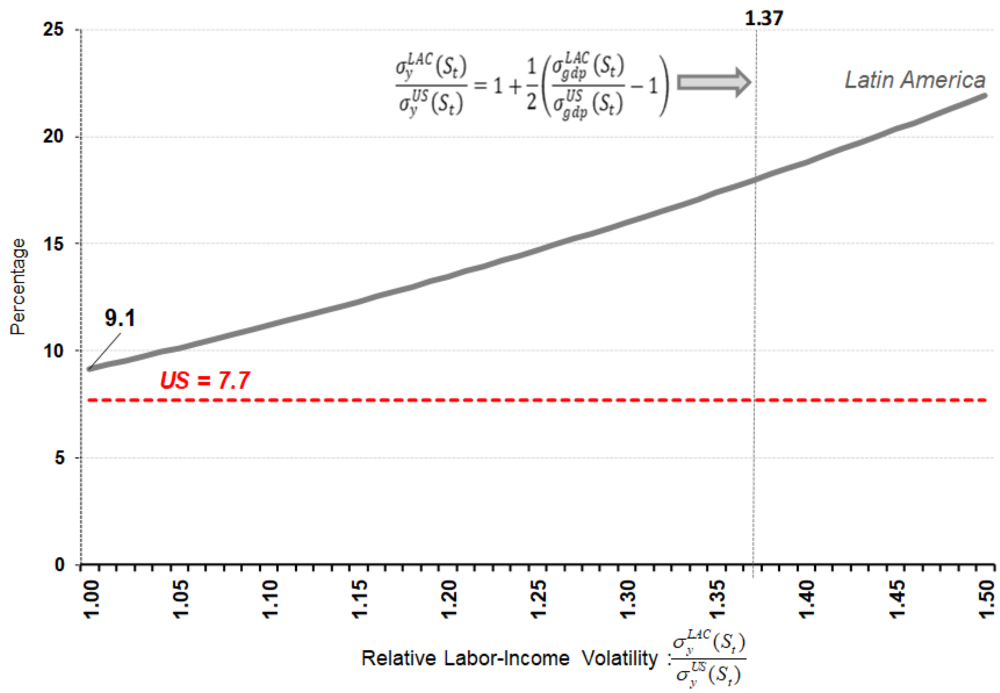
Table 5: Welfare Effects of Eliminating the Business Cycle with Idiosyncratic Risk
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)

		(E1) With Benchmark values: $\sigma_y(L)=0.26$ and $\sigma_y(H)=0.10$				(E1) With Benchmark values: $\sigma_y(L)=0.26$ and $\sigma_y(H)=0.10$	
Country	***E[u(S)]***	***Δû***	***Welfare Effects Δc***	***Country***	***E[u(S)]***	***Δû***	***Welfare Effects Δc***
United States	2.4	0.32	**7.7**	*Argentina*	1.5	0.40	**9.6**
Australia	2.1	0.32	**7.7**	*Bolivia*	0.8	0.35	**8.4**
Austria	2.5	0.33	**7.9**	*Brazil*	2.1	0.38	**9.1**
Belgium	2.3	0.33	**7.8**	*Chile*	2.0	0.41	**9.7**
Denmark	2.0	0.34	**8.2**	*Colombia*	1.7	0.34	**8.1**
Finland	2.8	0.34	**8.2**	*Costa Rica*	1.7	0.37	**8.9**
France	2.5	0.32	**7.7**	*Dominican Republic*	2.9	0.39	**9.5**
Greece	3.3	0.33	**8.0**	*Ecuador*	1.8	0.34	**8.2**
Ireland	2.8	0.34	**8.2**	*El Salvador*	1.1	0.39	**9.5**
Italy	2.9	0.33	**8.0**	*Guatemala*	1.3	0.33	**8.0**
Japan	3.5	0.34	**8.1**	*Honduras*	1.2	0.36	**8.5**
Netherlands	2.2	0.34	**8.1**	*Mexico*	1.9	0.35	**8.5**
New Zealand	1.3	0.34	**8.2**	*Nicaragua*	(0.1)	0.49	**11.7**
Norway	2.6	0.33	**7.9**	*Panama*	2.7	0.40	**9.7**
Portugal	3.2	0.37	**8.8**	*Paraguay*	1.1	0.37	**9.0**
Spain	3.1	0.34	**8.1**	*Peru*	0.8	0.39	**9.4**
Sweden	1.6	0.33	**8.0**	*Uruguay*	1.2	0.40	**9.7**
United Kingdom	2.3	0.33	**7.9**	*Venezuela, RB*	1.4	0.34	**8.3**
Developed Countries	***2.5***	***0.33***	***8.0***	***Latin America***	***1.5***	***0.38***	***9.1***

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36, \gamma = 1$ (log-utility), $\beta = 0.96$, and total depreciation rate is 0.06. The welfare effects (Δc) is calculated with (32) and $\hat{\mu}$ with (31).Simple average by country group.

Even though there are not reliable estimates of labor income volatility in Latin American countries relative to the US values ($\sigma_y^{LAC}(S)/\sigma_y^{US}(S)$), a sensitivity exercise captures the equivalent variation in per capita consumption for different values of the labor income volatility in Latin America relative to the US benchmark (Figure 5). As an extreme case, the cost of real fluctuation could represent more than 20 percent of per capita consumption in Latin America, if its labor income volatility were 50 percent higher than in US. Let's, just for a comparison purpose, assume that the ratio of the labor-income volatility between Latin America and US is half of the difference in their relative per-capita GDP volatility. This would result in an average welfare cost of almost 18 percent in Latin America, if the labor-income volatility in this region were about 37 percent higher than the one in US. Although, there does not exit an aggregate estimate of the relative magnitude, the welfare cost is increasing in the labor-income volatility, crossing the 10 percent of per-capita annual consumption as soon as labor-income volatility is just 5 percent higher than the one in US.

Figure 5: Sensitivity Analysis Assuming that the Ratio Between the Labor Income Volatility in Latin American Region and US Ranges from 1 to a Factor of 1.5
(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)



Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36, \gamma = 1$ (log-utility), $\beta = 0.96$, and total depreciation rate is 0.06. The welfare effects (Δc) is calculated with (32) and $\hat{\mu}$ with (31).Simple average by country group. "LAC" stands for "Latin America".

4 Conclusions

Table 6 compares the equivalent variation in consumption level that has the same positive effects of eliminating the real volatility for the three models analyzed (based on the same parameter values). The baseline (Lucas) specification shows the lowest results for the welfare cost by country, while the greatest change in consumption is obtained in the general equilibrium model. The main reason behind this finding is that the elimination of business cycle in the second alternative model is growth-enhancing by increasing the adjusted consumption growth rate ($\hat{\mu}$).

Table 6: Comparative Results on the Welfare Effects of Eliminating the Macroeconomic Volatility *(Equivalent variation of per capita annual household consumption in percentage, 1961-2006)*

	Welfare Effects Δc				Order of magnitud higher w.r.t. the baseline model		
	Baseline (Lucas) Model	Alternative (I): Stochastic trend in the consumption process	Alternative (II): GE model with an uninsurable idiosyncratic labor risk		Alternative I	Alternative (II)	
			$\sigma_y^{US} = \sigma_y^{DC} = \sigma_y^{LAC}$	$\frac{\sigma_y^{LAC}}{\sigma_y^{US}} = 1.37$		Same labor income volatlility	$\frac{\sigma_y^{LAC}}{\sigma_y^{US}} = 1.37$
United States	0.006	0.3	7.7	n/a	1.7	3.1	n/a
Latin America	*0.059*	*3.0*	*9.1*	*17.9*	1.7	2.2	2.5
Argentina	0.094	5.1	9.6	18.8	1.7	2.0	2.3
Bolivia	0.015	1.1	8.4	16.4	1.9	2.8	3.0
Brazil	0.040	3.3	9.1	17.7	1.9	2.4	2.6
Chile	0.114	6.1	9.7	19.1	1.7	1.9	2.2
Colombia	0.010	0.8	8.1	15.9	1.9	2.9	3.2
Costa Rica	0.047	2.8	8.9	17.5	1.8	2.3	2.6
Dominican Republic	0.087	5.3	9.5	18.6	1.8	2.0	2.3
Ecuador	0.011	1.0	8.2	16.1	2.0	2.9	3.2
El Salvador	0.061	-	9.5	18.5	-	2.2	2.5
Guatemala	0.005	0.5	8.0	15.6	2.0	3.2	3.5
Honduras	0.022	1.5	8.5	16.7	1.8	2.6	2.9
Mexico	0.026	1.5	8.5	16.6	1.8	2.5	2.8
Nicaragua	0.171	-	11.7	23.0	-	1.8	2.1
Panama	0.097	6.3	9.7	19.0	1.8	2.0	2.3
Paraguay	0.031	-	9.0	17.6	-	2.5	2.8
Peru	0.073	-	9.4	18.3	-	2.1	2.4
Uruguay	0.101	-	9.7	18.9	-	2.0	2.3
Venezuela, RB	0.053	3.8	8.3	18.2	1.9	2.2	2.5
Other DC	*0.010*	*0.8*	*8.0*	*n/a*	1.9	2.9	*n/a*

Source: Author's calculations based on WDI, ECLAC and PWT data.
Note: Basic Assumptions: $\alpha = 0.36$,$\gamma = 1$ (log-utility),$\beta = 0.96$, and total depreciation rate is 0.06. Simple average by country group. In Alternative (I): Countries where the estimate of $\hat{g}$ is not statistically significant at the 10 percent level are not listed. "DC" stands for "developed countries" and "LAC" for "Latin America".

Under the Lucas framework, real volatility is not relevant in terms of welfare for all developed countries and Latin American economies. However, the welfare effects in both Latin America and developed countries are on average more than two-orders of magnitude higher those estimated with the baseline model, even without taking into account the most recent international financial crisis. In sum, the magnitude of the welfare cost of macroeconomic volatility is framework-dependent, leaving unanswered the question of whether more countercyclical policies would be needed to reduce the macroeconomic fluctuations in most developing countries.

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